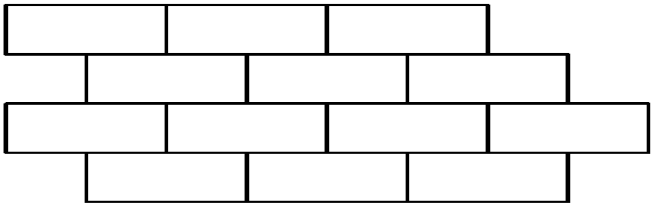
Tessellations

Katherine Sheu

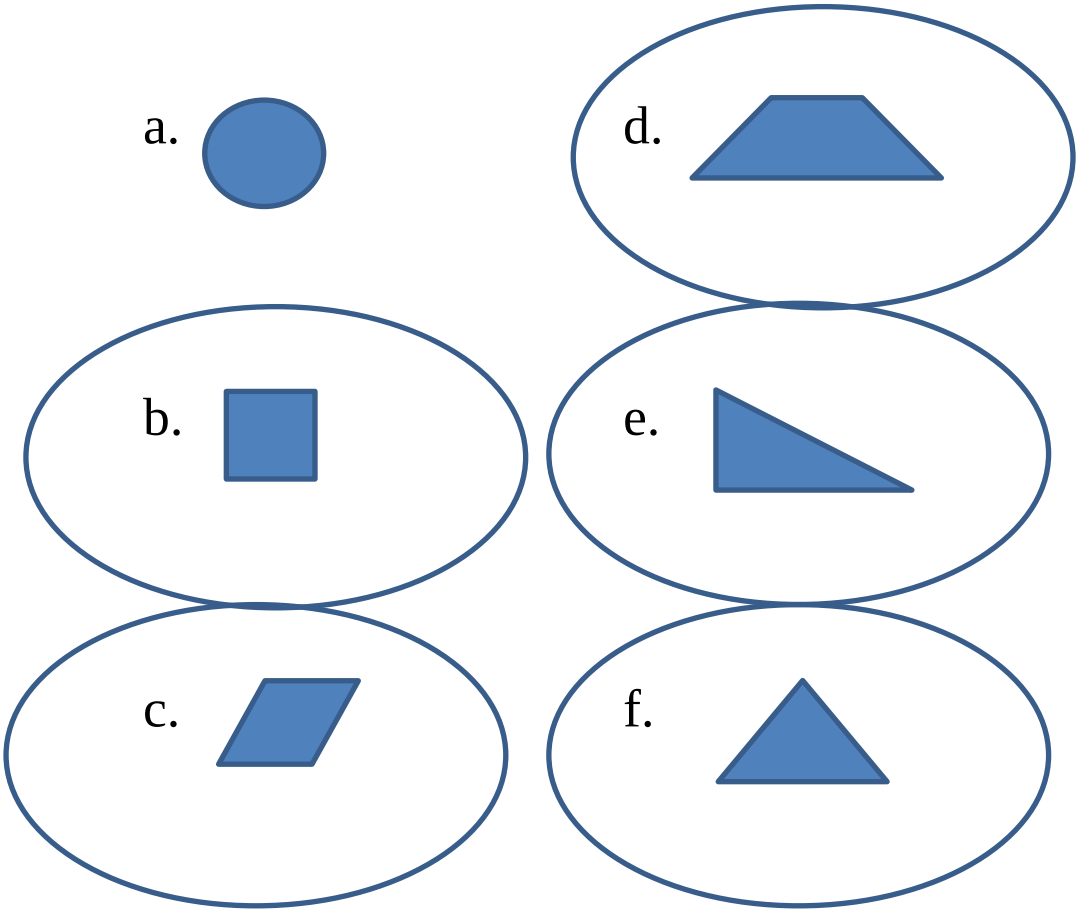
A **tessellation** is created when a shape is repeated over and over again to cover the plane without any overlaps or gaps.

1. The picture below can be extended to a tessellation of the plane. Add a few blocks to continue the tessellation.



2. **What shape is used in this tessellation?**
   Solution: A rectangle

2. Which of these shapes can be used to make a tessellation?



How do you determine whether a shape can form a tessellation?

Solution: By seeing if you can repeat it over and over again without getting any overlaps or gaps. I.e. By seeing if you can repeat the shape in such a way as to get a full turn at a vertex.

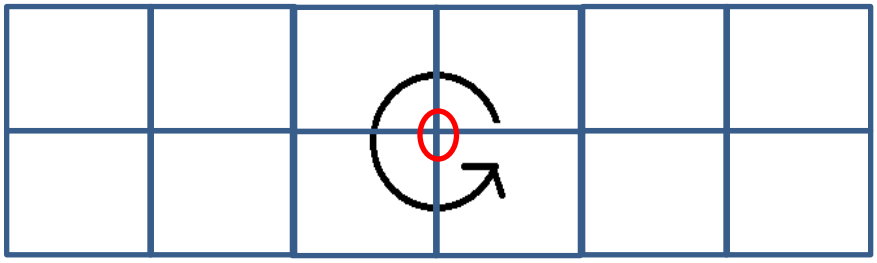
Tessellations by One Regular Polygon

A **regular polygon** is a shape with **all sides equal** and **all angles equal**. Some regular polygons tessellate. Others do not.

3. Which of the shapes below are regular polygons? Remember, to be regular, a polygon must have all equal sides AND all equal angles.

A **vertex** of a tessellation is a point where three or more corners of the tessellating shapes are joined.

For example, these squares fit together to make one full turn at a vertex, with no overlapping parts and no gaps. The square is therefore a regular polygon that tessellates.



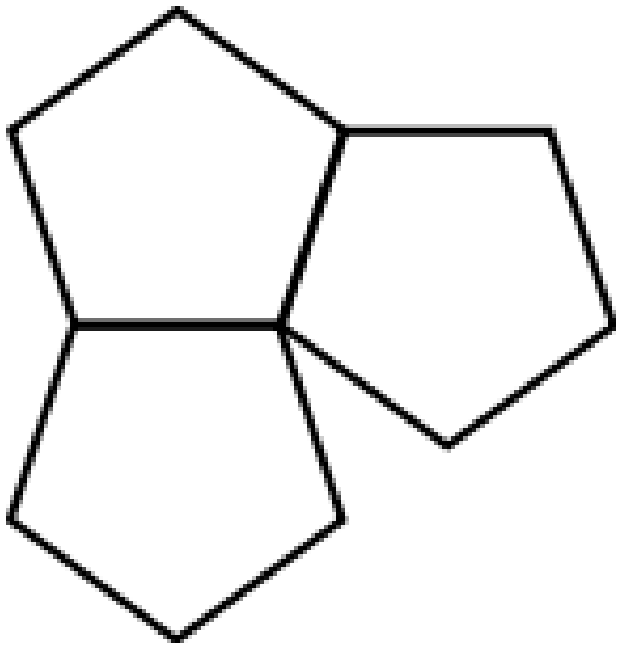
3a. Look at the figure above. Draw a circle around any vertex of this tessellation.

3b. Count how many squares the circle you drew intersects.

Solution: 4

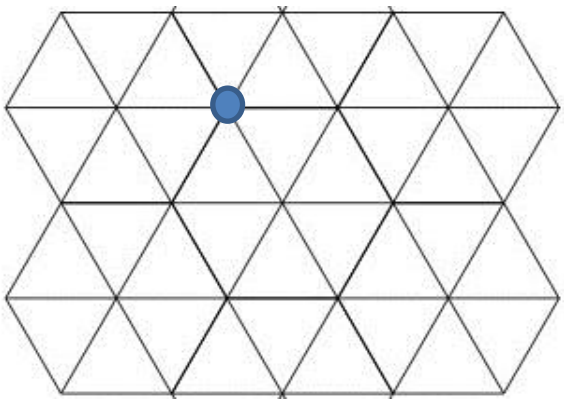
Use your pentagons to try to make a tessellation. Share what you have made with a neighbor.

Explain why the arrangement of regular pentagons below is not a tessellation.



Solution: Because there is no way to put the pentagons so that you can complete a full turn at a vertex. By using three pentagons, you will always get a gap, and by using any more, you will get an overlap.

Here is a tessellation by regular triangles.



4b. Draw a circle around a vertex in this tessellation.

4c. How many regular triangles are needed to complete one full turn at a vertex of the tessellation so that there are no gaps and no overlapping parts?

Solution: 6

5a. Use your shapes to see if regular hexagons tessellate. Why or why not?

Solution: They do because you can create a repeating pattern that will have no overlap or gaps. In particular, you can put the hexagons together so that they complete a full turn.

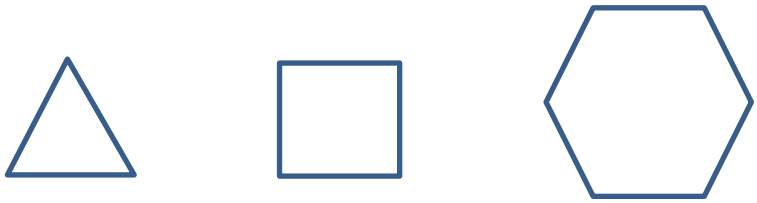
5b. Look at the figure you have built with your hexagons. How many regular hexagons are needed to complete one full turn at a vertex of the tessellation so that there are no gaps and no overlapping parts?

Solution: 3

Activity: Tessellations by Several Regular Polygons

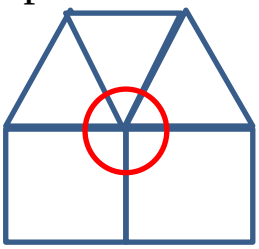
Now we will look at tessellations involving several types of regular polygons.

We are going to use only these three shapes in this activity:



Find as many ways as you can to fit these shapes to make a full turn at the vertex of the tessellation.

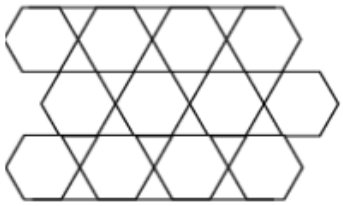
Here's an example:

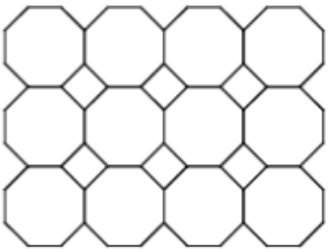


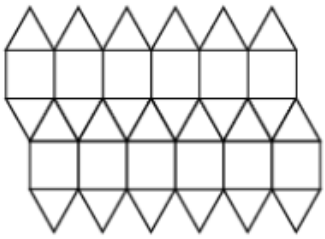
Two squares and three regular triangles will make a full turn at the vertex of this tessellation.

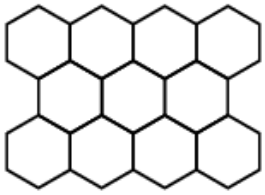
6. How many ways can you find? Share your answer with a neighbor.

Below are the 9 different ways to combine squares, hexagons, and triangles to make tessellations of the plane. Circle which ones of these you found:

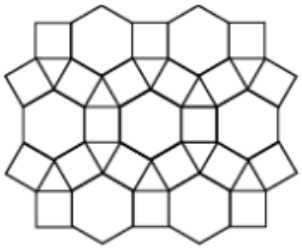
a) 

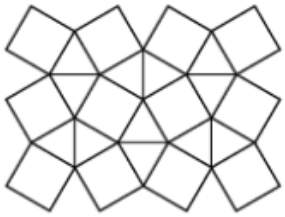
b) 

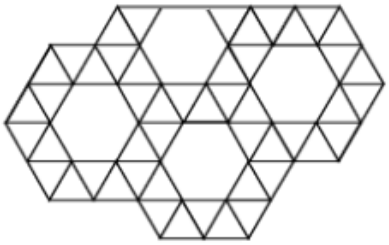
c) 

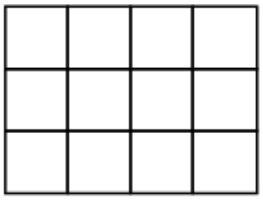
d) 

e) 

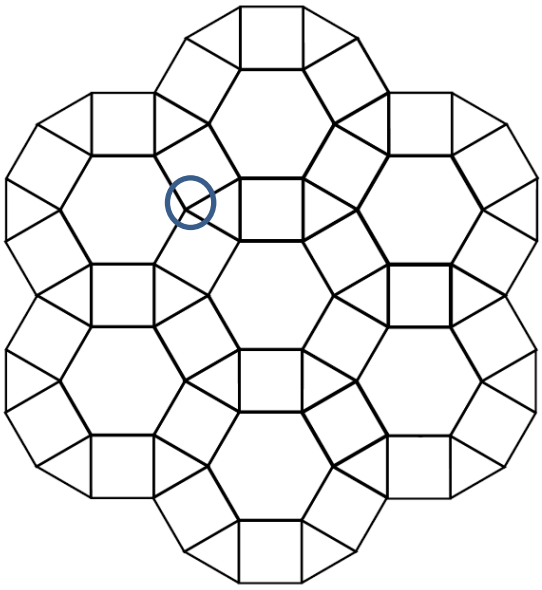
f) 

g) 

h) 

i) 

Now let's take another look at some of the tessellations we may have constructed in the activity.



7a. Draw a circle around any vertex in this tessellation.

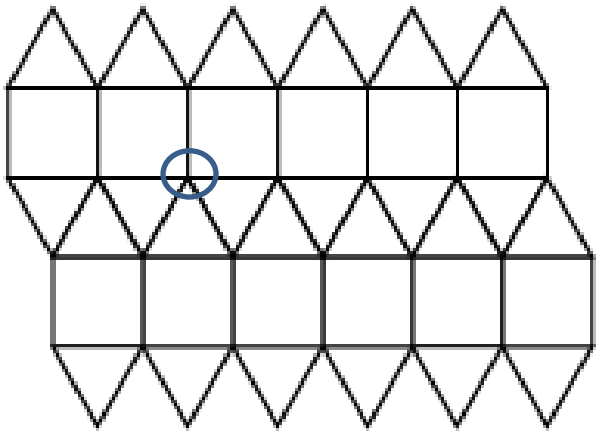
7b. Color the shapes that join at the vertex you circled, using one color for each type of shape.

7c. Which shapes and how many of each shape are needed to complete one full turn at a vertex of the tessellation?

Shape	**# of that shape**
Triangle	**1**
Square	**2**
Hexagon	**1**

7d. Repeat the process with another vertex in this tessellation.

Here is another tessellation.



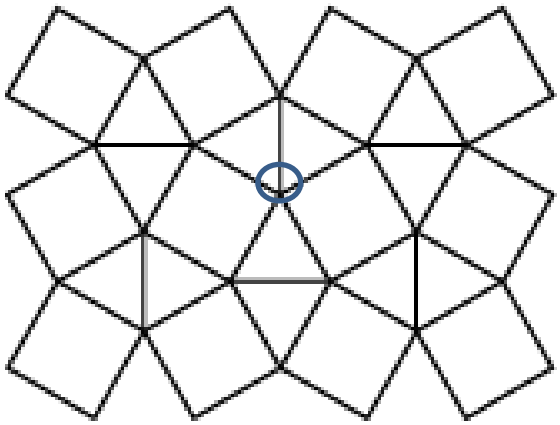
8a. Draw a circle around any vertex in this tessellation.

8b. Color the shapes that join at the vertex you circled, using one color for each unique shape.

8c. Which shapes and how many of each shape are needed to complete one full turn at a vertex of the tessellation?

Shape	**# of that shape**
Square	**2**
Triangle	**3**

Here is another tessellation.



9a. Draw a circle around any vertex in this tessellation.

9b. Color the shapes that join at the vertex you circled, using one color for each type of shape.

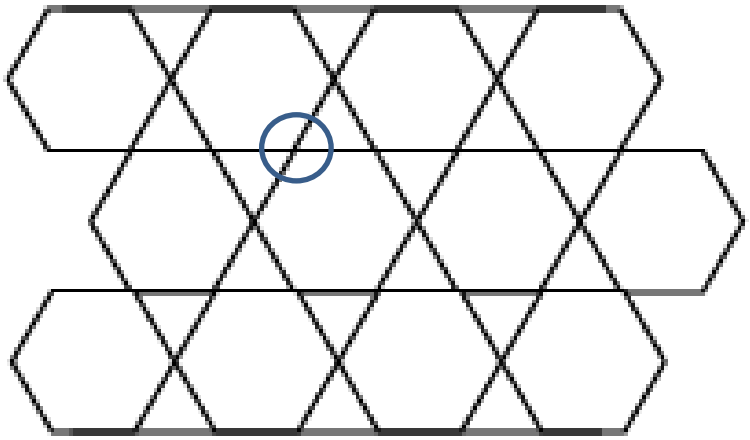
9c. Which shapes and how many of each shape are needed to complete one full turn at a vertex of the tessellation?

Shape	# of that shape
Triangle	3
Square	2

9d. Compare your answer for #8c with your answer for #9c. What do you observe?

Solution: They both have the same number of triangles and squares at a vertex.

Here is another tessellation.



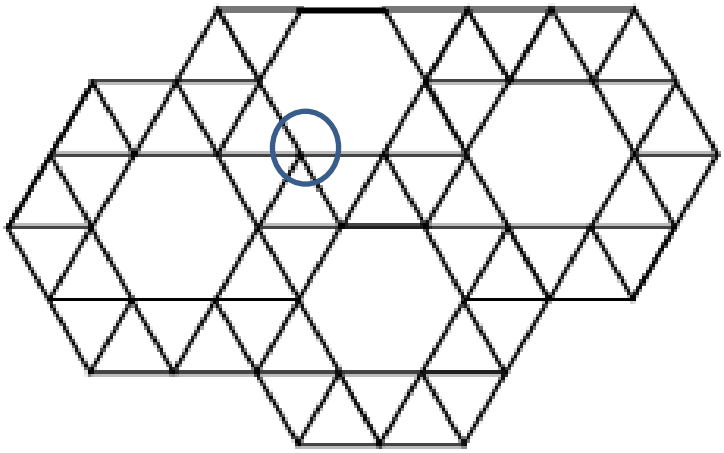
10a. Draw a circle around any vertex in this tessellation.

10b. Color the shapes that join at the vertex you circled, using one color for each type of shape.

10c. Which shapes and how many of each shape are needed to complete one full turn at a vertex of the tessellation?

Shape	# of that shape
Hexagon	2
Triangle	2

Here is another tessellation.



11a. Draw a circle around any vertex in this tessellation.

11b. Color the shapes that join at the vertex you circled, using one color for each type of shape.

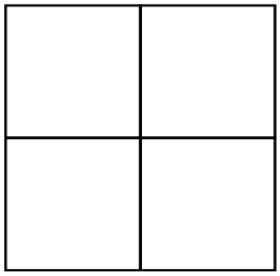
11c. Which shapes and how many of each shape are needed to complete one full turn at a vertex of the tessellation?

Shape	# of that shape
Triangle	4
Hexagon	1

12. Let's summarize what we have so far.

Question #	# of Regular Triangles	# of Squares	# of Regular Hexagons
#3b	0	4	0
#4c	6	0	0
#5b	0	0	3
#7c	1	2	1
#8c	3	2	0
#9c	3	2	0
#10c	2	0	2
#11c	4	0	1

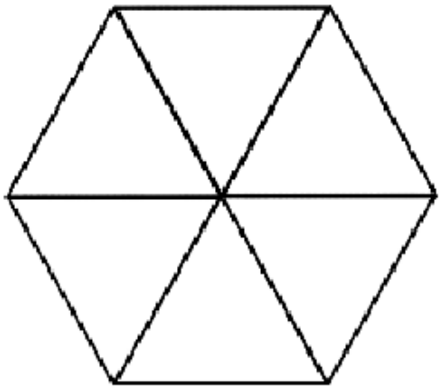
13a. If four squares complete one full turn (360 degrees) at the vertex of the tessellation, what is the angle measurement of each corner in a square?



Solution: 90 degrees (360/4=90)

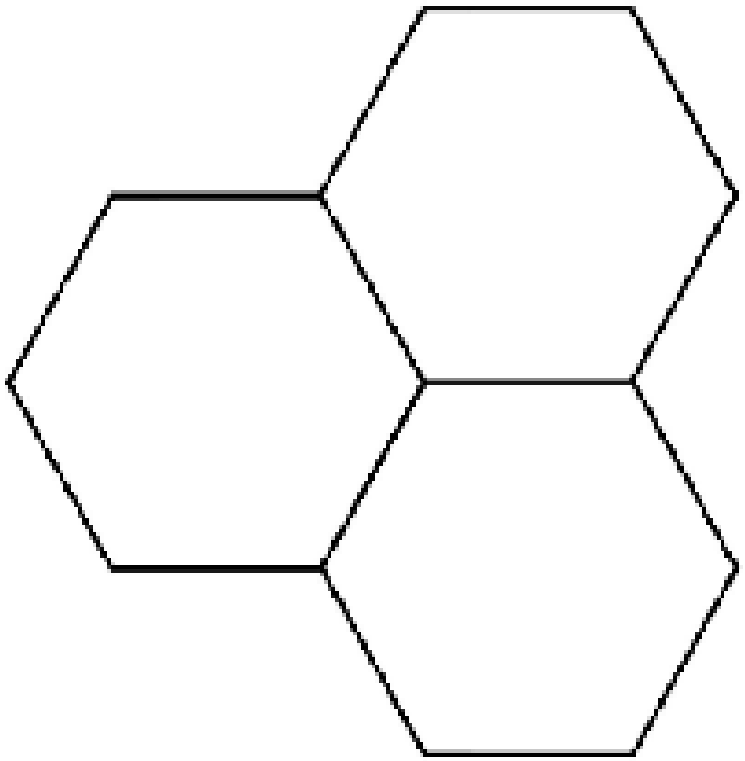
13b. If six triangles complete one full turn (360 degrees) at the vertex of the tessellation, what is the angle measurement of each corner in a regular triangle?

Solution: 60 degrees (360/6=60)



13c. If three hexagons complete one full turn (360 degrees) at the vertex of the tessellation, what is the angle measurement of each corner in a regular hexagon?

Solution: 120 (360/3=120)



14. Look back to the chart you made in #12. Use what you know from #13 to find the sum of the angle measurements at the vertices of each tessellation.

Question	# of Regular Triangles	# of Squares	# of Regular Hexagons	Sum of Angle Measurements at the Vertex
#3	0	4	0	360
#4	6	0	0	360
#5	0	0	3	360
#7	1	2	1	360
#8	3	2	0	360
#9	3	2	0	360
#10	2	0	2	360
#11	4	0	1	360

15. What do the vertices of all these tessellations have in common?

The sum of angle measurements at any vertex is 360 degrees!