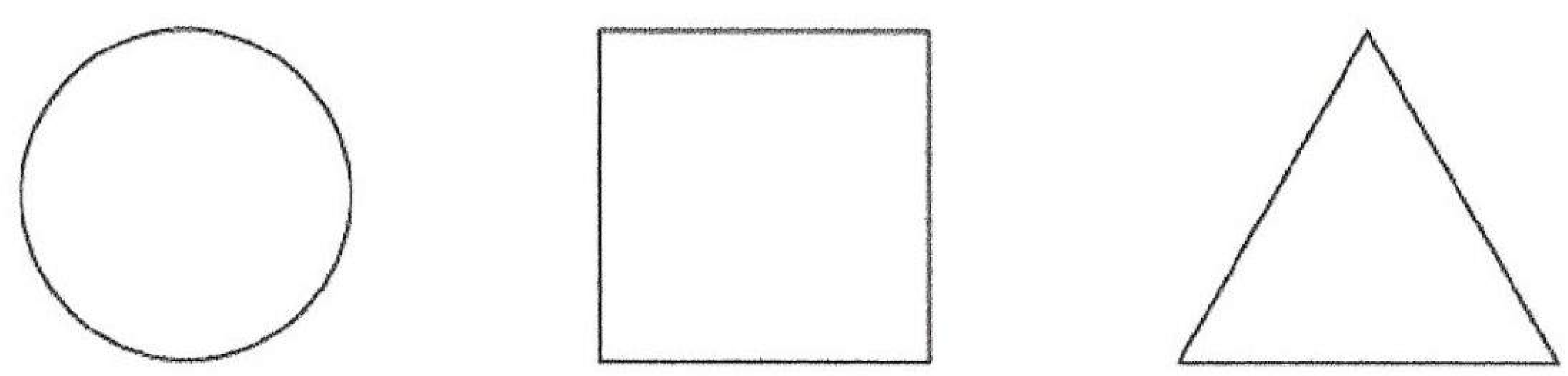
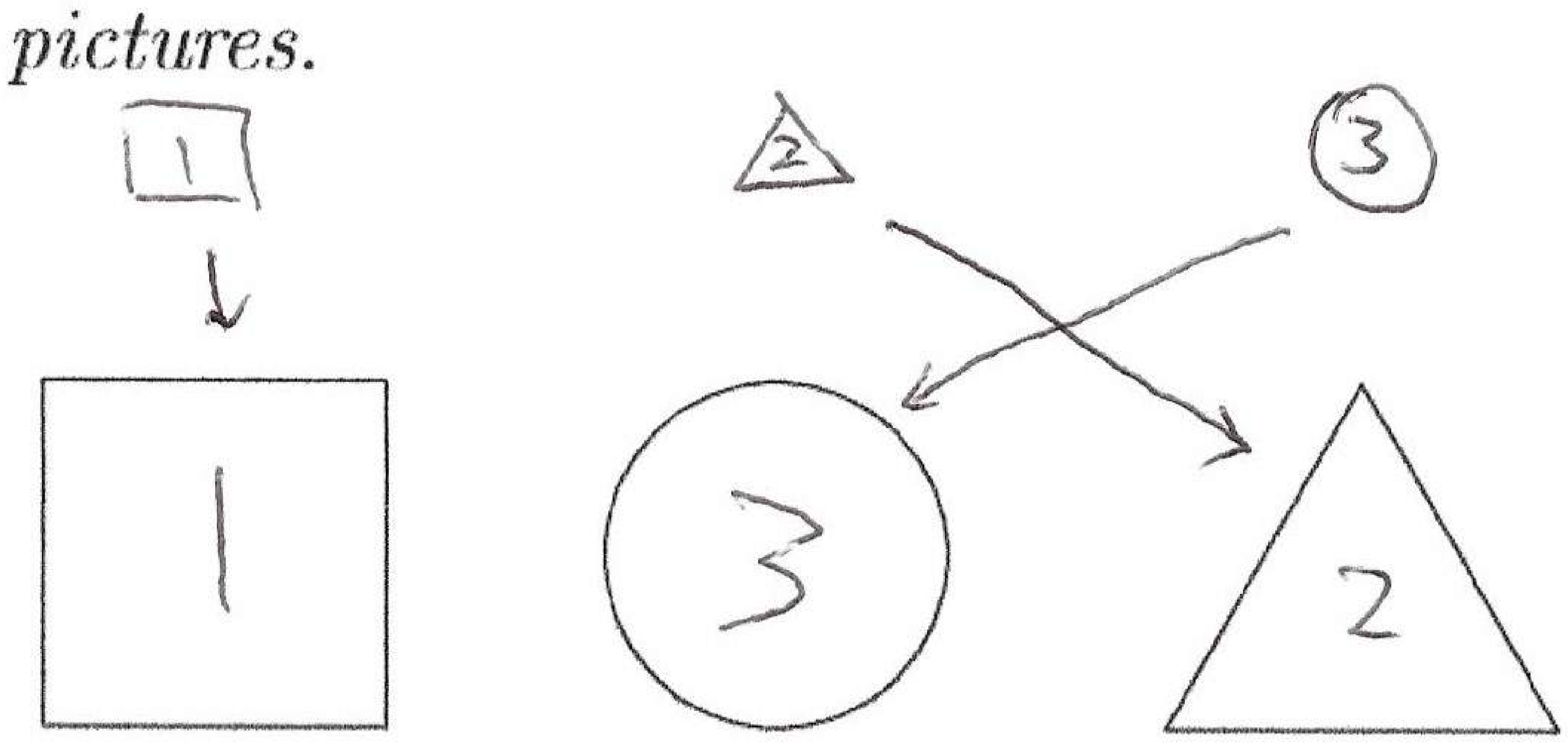
Then the permutation

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

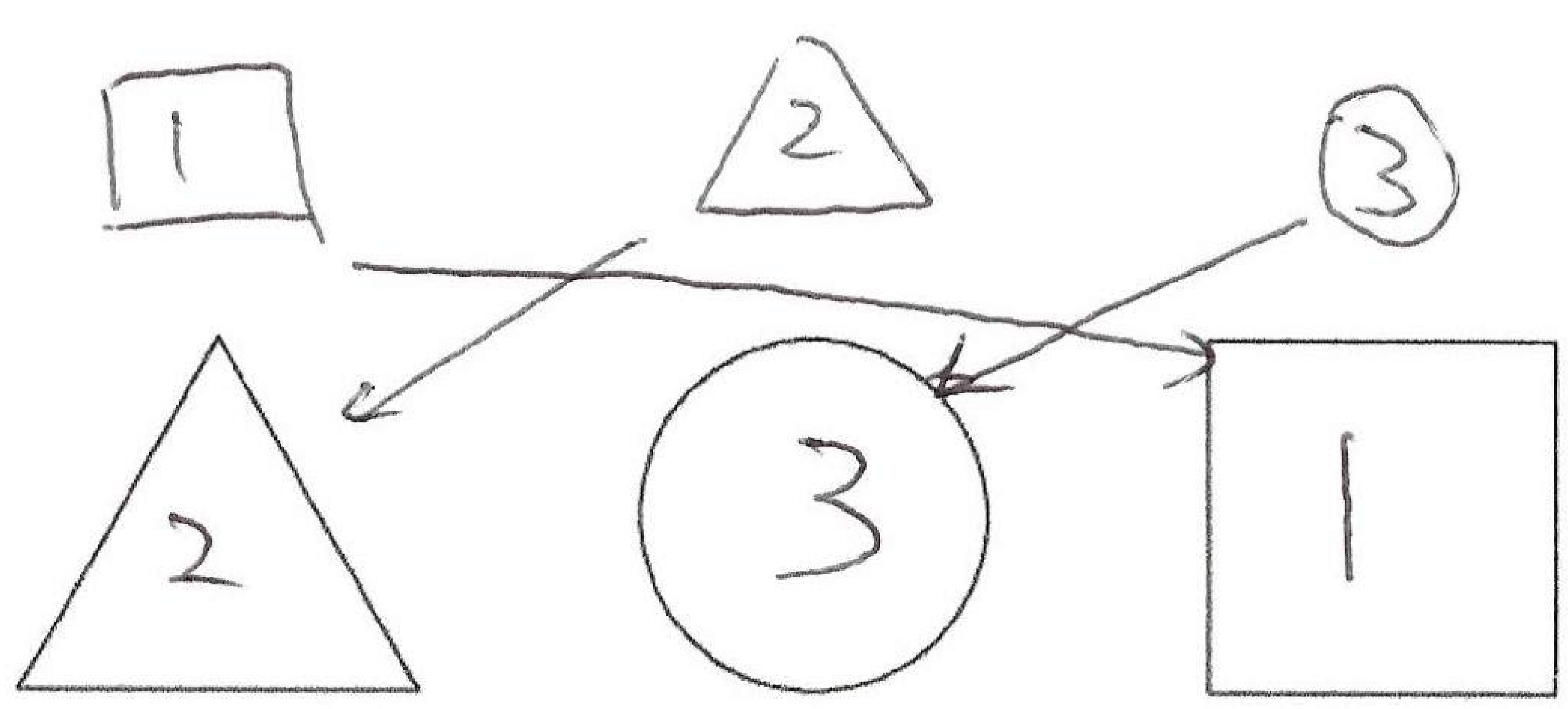
will reshuffle the figures into the following order.



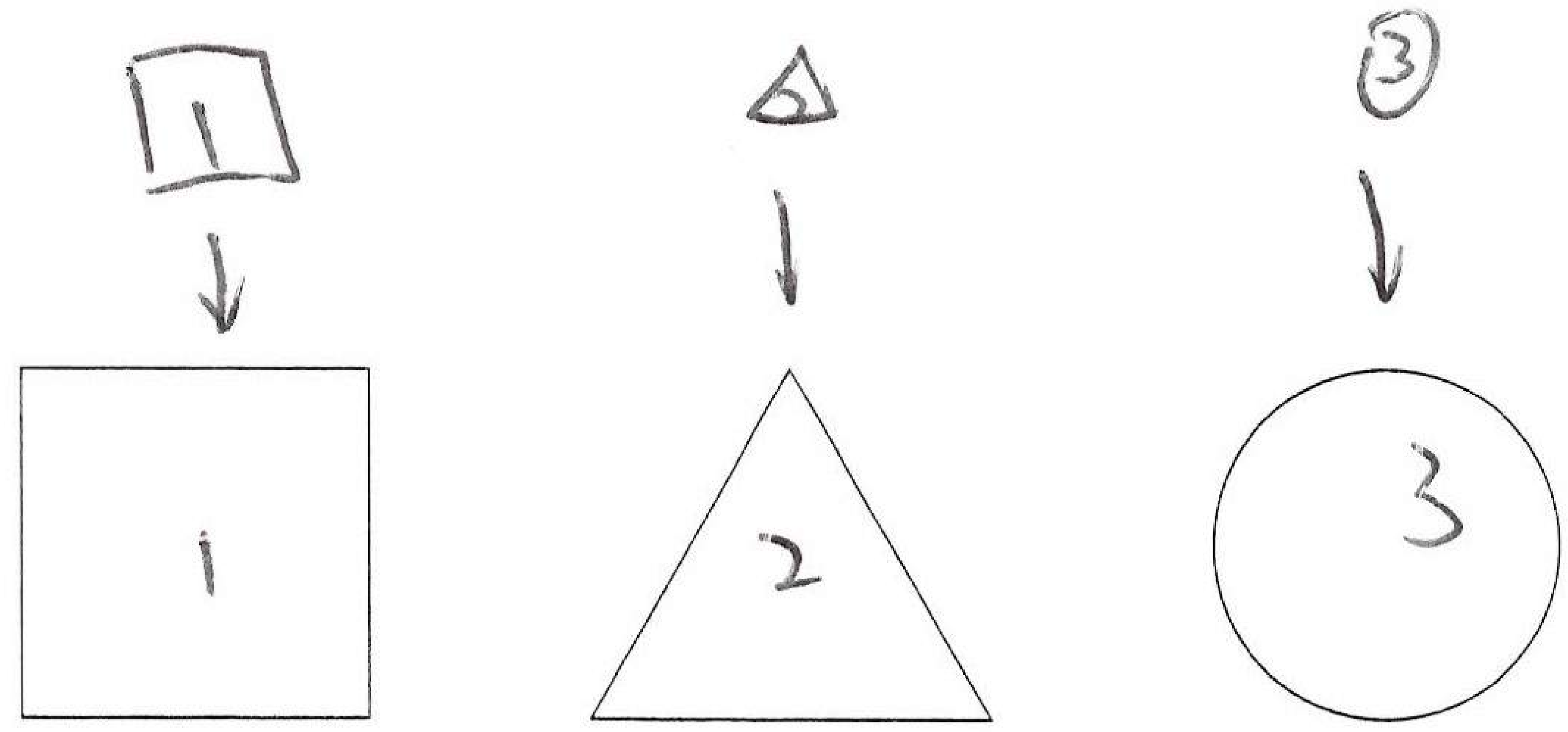
Problem 5 *For the original order of figures given on page 5, write down the permutations that correspond to the following pictures.*



$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

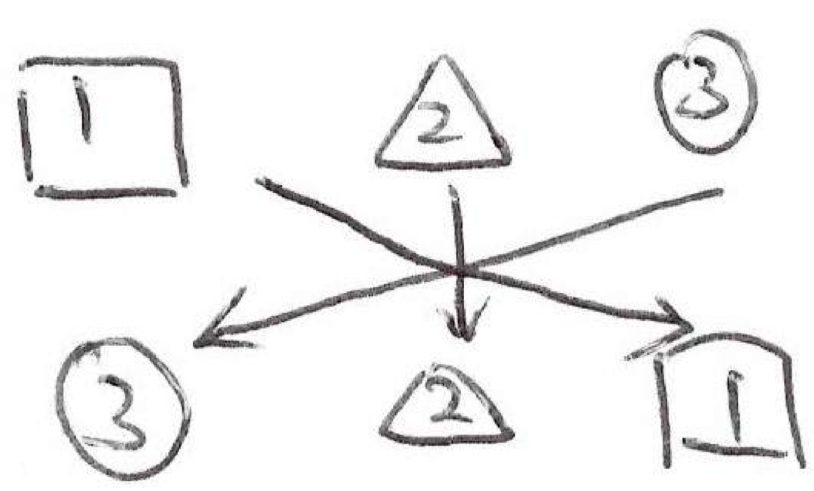
Note that the last permutation does not reshuffle anything at all. Permutations of this kind typically denoted as e and called *trivial.* A trivial permutation is still a permutation, and an important one!

Problem 6 *Write down the trivial permutation for* $n = 5$.

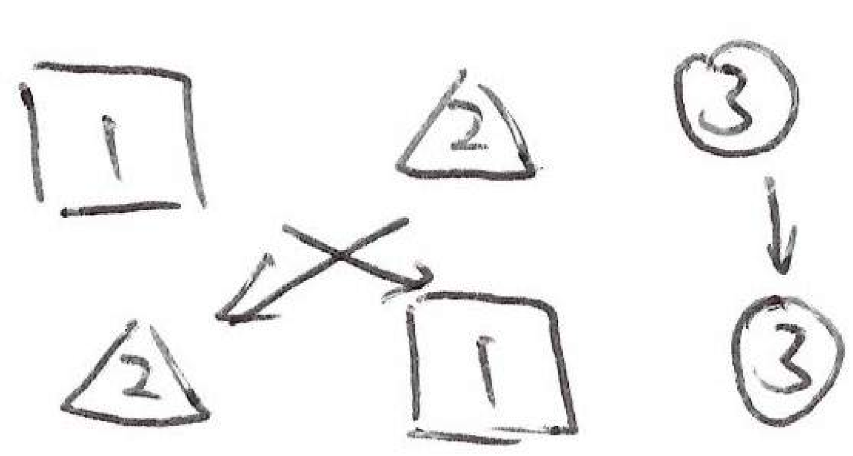
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

Problem 7 *For the original order of figures given on page 5, draw the figures in the orders prescribed by the permutations below. Use the space to the right of a permutation to draw the corresponding picture.*

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$



Recall that $n! = 1 \times 2 \times \ldots \times (n-1) \times n$. For example, $3! = 1 \times 2 \times 3 = 6$.

Problem 8 *Compute* 5!

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Problem 9 *How many permutation of four elements are there?*

For the first place, we have 4 choices.
- - - 2nd - - , - - - 3 - - - .
- - - 3nd - - , - - - 2 - - - .
- - 4th - - , - - - 1 choice.

Thus, we have $4 \times 3 \times 2 \times 1 = 4! = 24$ permutations

Problem 10 *How many permutation of n+1 elements are there?*

Similarly to what we did above, for the 1st place, $(n+1)$ choices of elements

$\vdots$ $\vdots$

1st 1 choices

In total, there will be $(n+1) \cdot n(n-1) \cdot \ldots \cdot 1 = (n+1)!$ permutations

Problem 11 *Write down a permutation of four elements.*

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

Note: There are many permutations (24, to be precise).

Problem 12 *Write down a permutation of four elements that keeps the third element in place.*

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$$

Note that there will be $3 \times 2 \times 1 \times 1 = 6$ permutations satisfying this standard. (The 1st place has 3 choices of element, 2nd has 2, and 3rd has 1.)

Find the number of permutations of four elements that keep the third element in place.

The number $= 3 \times 2 \times 1 \times 1 = 6$

It is possible to combine, or *multiply*, permutations. For example, let us apply the permutation

$$\delta = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

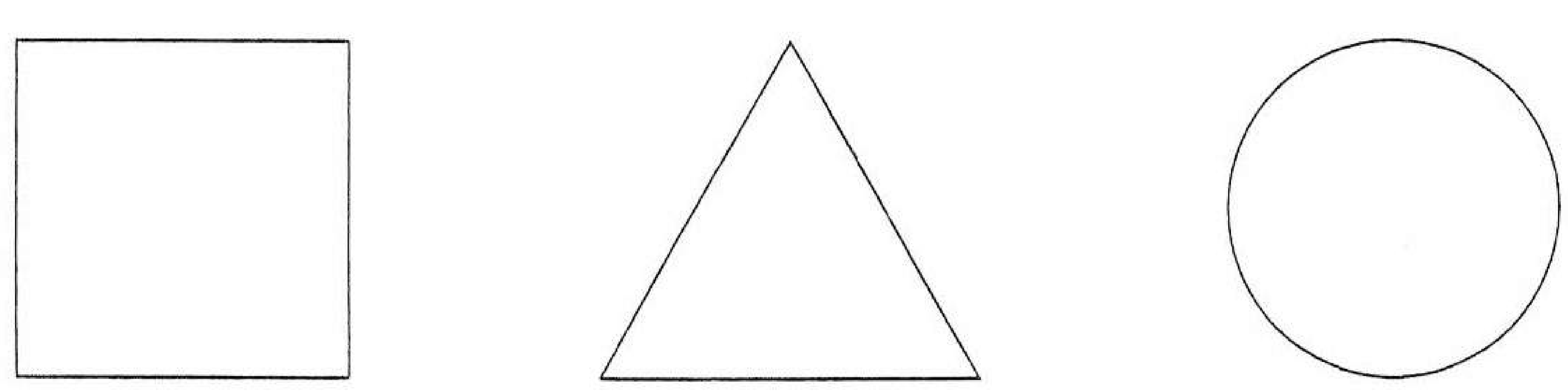
to the marbles already reshuffled by the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$$

The permutation δ switches the first and second elements, so

$$\delta \circ \sigma = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}.$$

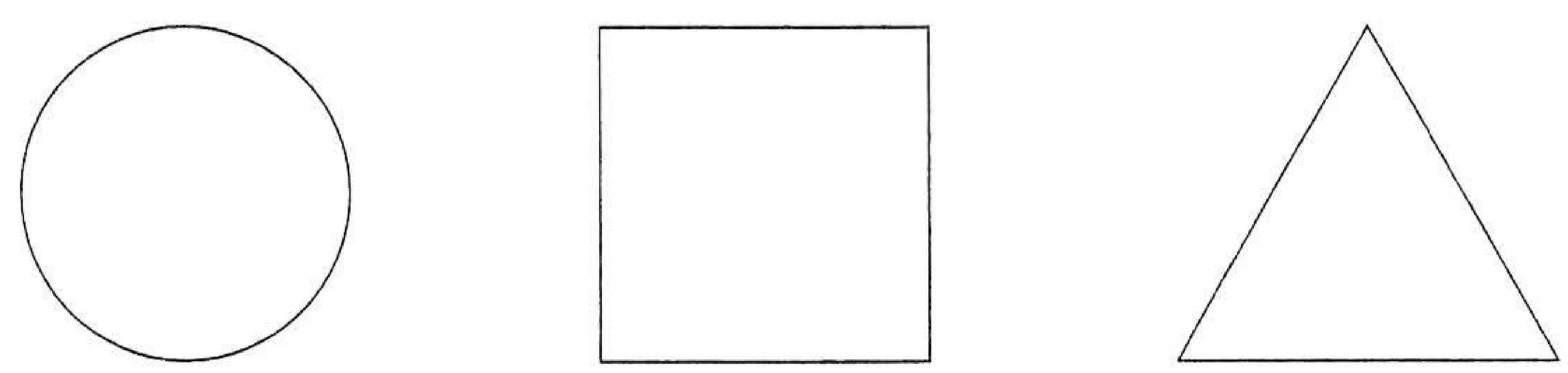
Let us take another look at the above computation using the figures from page 5. Originally, the set of the figures is ordered as follows.



The permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

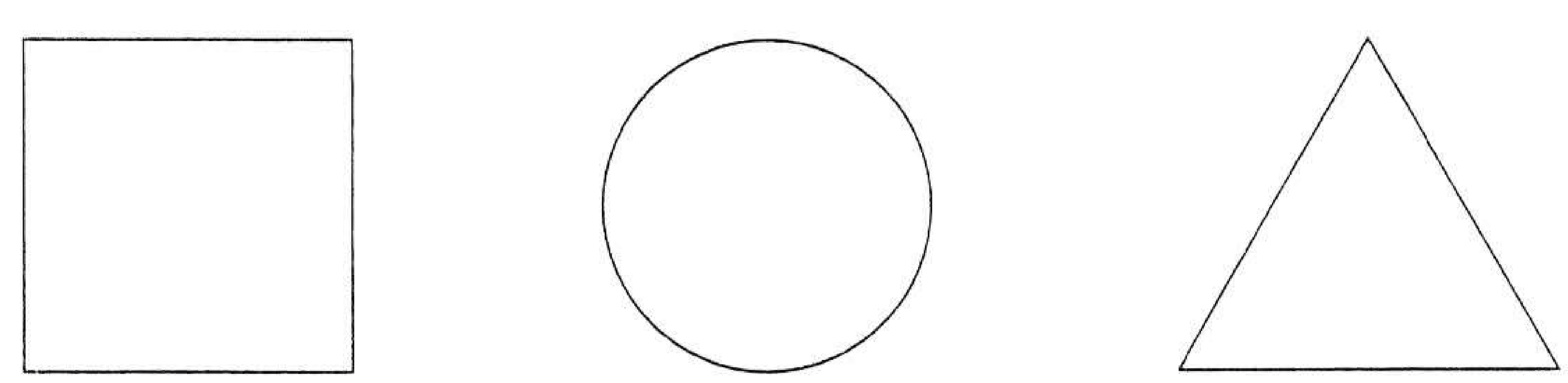
produces the picture below.



The permutation

$$\delta = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

applied to the latter configuration gives us the following.



Comparing the last picture to the original gives us the answer.

$$\delta \circ \sigma = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

Note that in the product $\delta \circ \sigma$ of permutations, it is the one on the right, σ, that acts first on the set it permutes!