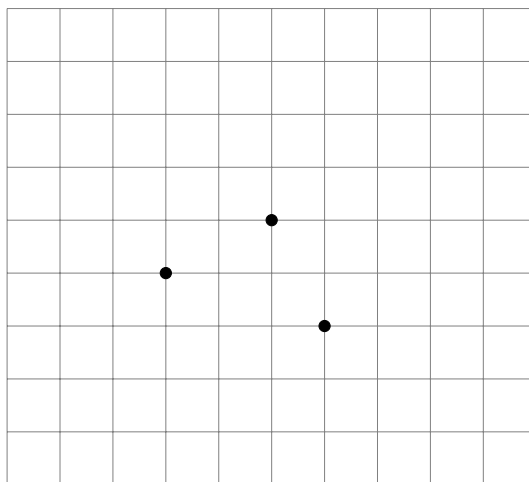


Oleg Gleizer
oleg1140@gmail.com

Review Session

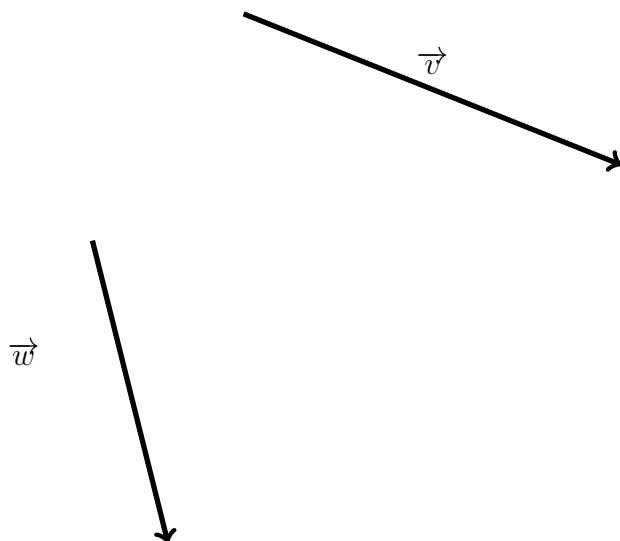
Problem 1 *Three points are marked on the grid below. Draw the fourth point so that the four points and the lines connecting them form a parallelogram. Find all the possible solutions!*



Problem 2 *Use a compass and a ruler to divide the segment below into three parts of equal length.*



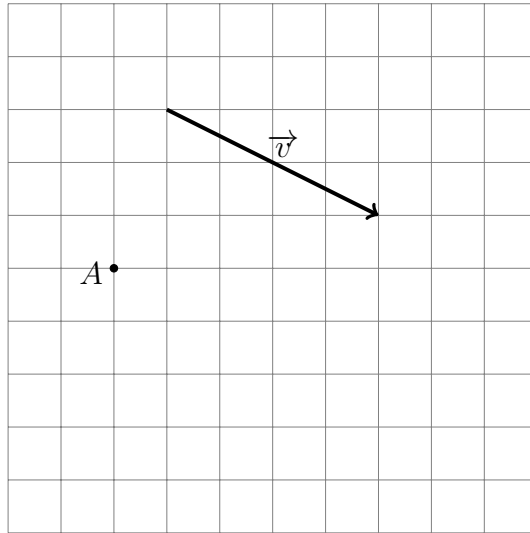
Problem 3 Use a compass and a ruler to construct the sum $\vec{v} + \vec{w}$ of the vectors \vec{v} and \vec{w} given below.



Problem 4 Prove that addition of vectors is commutative.

$$\vec{v} + \vec{w} = \vec{w} + \vec{v} \quad (1)$$

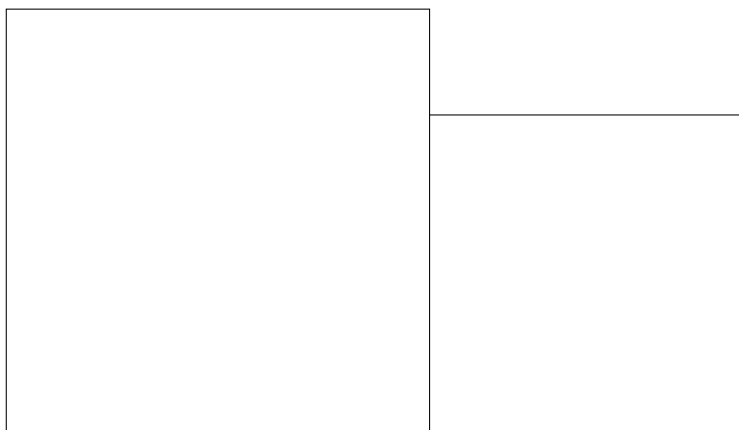
Problem 5 For the given vector \vec{v} and point A , construct the vector $\vec{w} = -1.5\vec{v}$ having A as its terminal point on the graph paper below.



Problem 6 For the given vectors \vec{v} and \vec{w} , construct the vector $\frac{2}{3}\vec{w} - .75\vec{v}$ on the graph paper below.



Problem 7 *Cut the two squares below into five parts such that one can make one square out of the parts, using all the parts and without overlapping. Use a compass and a ruler to draw all the necessary lines with maximal precision!*



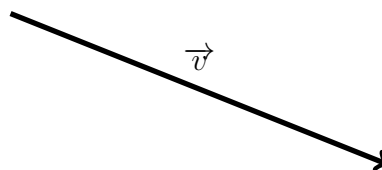
Problem 8 *Given the segment of length a , use a compass and ruler to construct a segment of length $\sqrt{2}a$ in the space below.*

a



Problem 9 *Use a compass and ruler to construct a segment of length $\sqrt{5}a$ in the space below where a is the length of the segment from Problem 8.*

Problem 10 Use a compass and a ruler to construct the vector $\vec{w} = \sqrt{5}\vec{v}$ for the vector \vec{v} given below such that point C is its initial point.

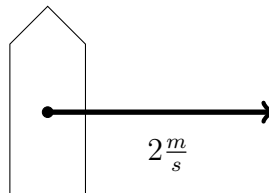


C •

Problem 11 *Give the definition of a rational number.*

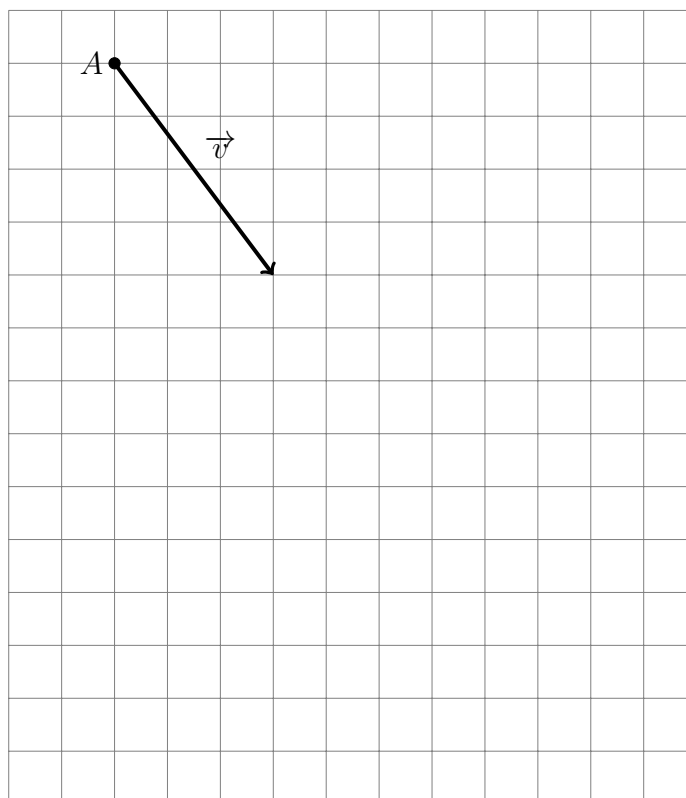
Problem 12 *Prove that $\sqrt{5}$ is not rational.*

Problem 13 *A man is crossing a river in a boat. The speed of the boat is five metres per second. The speed of the water in the river is two metres per second. In what direction should the man steer the boat, if he wants the vessel to move perpendicular to the banks? Please use a compass and a ruler to construct the velocity vector.*



The width of the river is $60\sqrt{21}$ meters. How long would it take the man to cross the river?

Problem 14 *There are no forces acting on the object A below. The current velocity of the body, in metres per second, is represented by the vector \vec{v} . Draw the position of the object three seconds later.*



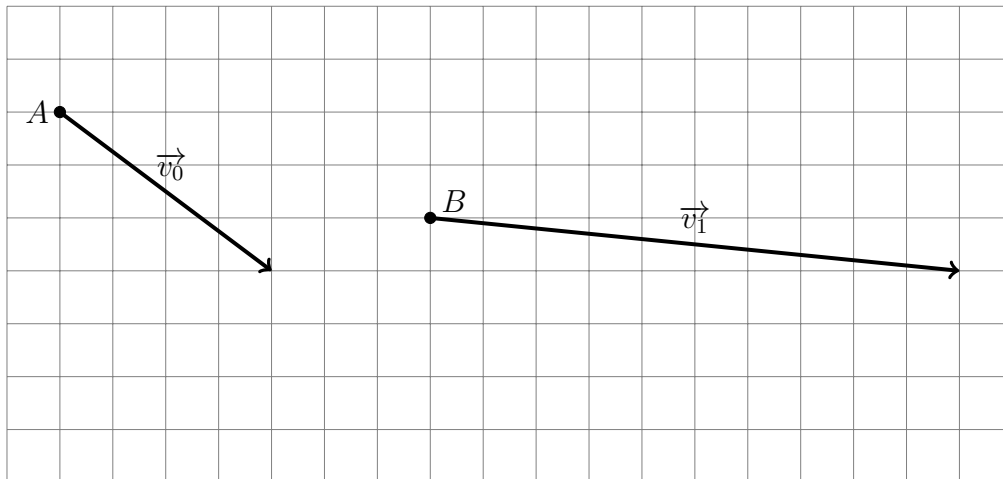
The sides of the grid squares on the picture above are ten metres long. Find the speed of the motion. (Please continue to the next page.)

$$v =$$

What distance would the object cover in a minute?

$$d =$$

Problem 15 *Moving with a constant acceleration, a body changes its position from point A to point B in one second. The velocities of the motion, in metres per second, are represented by the vectors \vec{v}_0 and \vec{v}_1 . Find the acceleration vector.*



Problem 16 *The General Electric F110-GE-129 turbofan engine powering the F-16 fighter jet generates 29,588 lbf (131.6 kN) of thrust.*



F-16

The aircraft, armed and loaded with fuel, weighs 26,500 lbs (12,000 kg). The free-fall acceleration near the Earth's surface is 9.8 m/s^2 (32.2 ft/s^2). Is a fully loaded F-16 capable of ascending vertically? Why or why not?

Problem 17 *There are four courses taught at some school. Students take the classes as follows.*

300 take Language Arts.

280 take Math.

285 take PE.

276 take Science.

192 take Language Arts and Math.

180 take Language Arts and PE.

169 take Language Arts and Science.

168 take Math and PE.

144 take Math and Science.

125 take PE and Science.

90 take Language Arts, Math, and PE.

80 take Language Arts, Math, and Science.

67 take Language Arts, PE, and Science.

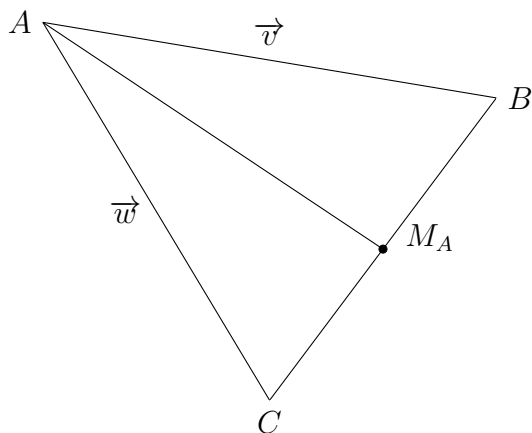
75 take Math, PE, and Science.

24 take all four.

Each student takes at least one of the classes.

How many students are there in the school?

Problem 18 Use vector algebra to prove that all the three medians of a triangle in the Euclidean plane intersect at one point that splits each of the medians in the ratio 2:1 counting from the vertices.



Problem 19 *Draw the scheme of a pulley with the advantage two. Explain how it works.*

Problem 20 *There are 64 cities in the country of Ostlund. Prove that at least three of them will have the same number of rainy days in September.*

Problem 21 *Prove that in any company there exist at least two people who have the same number of acquaintances.*

Problem 22 *In the sequence 1, 1, 2, 3, 5, 8, 13, ... every number (except for the first two) is a sum of the two preceding numbers.*

- *Can the sequence have two neighbouring even numbers?*

- *Can the sequence have two neighbouring numbers, both multiples of 13?*