

MATH KANGAROO (LEVEL 7-8)

LAMC INTERMEDIATE - 3/02/14

All problems come from old Math Kangaroo tests, however, the possible answers have been removed (to deter you from guessing). Solve the problems in any order, they are not in order of difficulty.

- (1) Find the sum $x + y$ if x and y fulfill the conditions of the following equation:

$$(x - y - 1)^2 + (x + y - 7)^2 = 0$$

- (2) A square piece of paper with dimensions $10\text{cm} \times 10\text{cm}$ has been cut into squares with area 25cm^2 each. Then each square is cut into two triangles. How many triangles are there?

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- (3) The numbers $1, 2, 3, \dots, 1022, 1023, 1024$ have been placed around a circle clockwise in the order given. We move around the circle clockwise and erase every other number until only one number is left. Which number will be the last one left if we erase number 1 first.
- (4) A square of a positive number is 500% greater than that number. What is the number?
- (5) What is the greatest number of elements that can be chosen from $\{1, 2, 3, 4, \dots, 25\}$ so that the sum of any two of them is not divisible by three? (Give a maximal such set.)

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- (6) A certain father is 52 years old, and his sons are 24 and 18 years old. How many years later will the age of the father be the same as the sum of the ages of both his sons?
- (7) All positive whole number which are equal to the product of their proper factors (factors which are not 1 or the whole number) are written in ascending order. What is the sixth number that will be written?
- (8) What is the ones digit of the product of all the primes less than 2014?
- (9) How many pairs of digits a, b from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ make the following equation true: $a \cdot b = 10 + a$.

- (10) There are seven cards in a box. Numbers from 1 to 7 are written on these cards, exactly one number on each card. The first sage takes 3 cards at random from the box and the second sage takes 2 cards. 2 cards are left in the box. Then, the first sage says to the second one: "I know the sum of the numbers of your cards is even." The sum of the numbers on the cards of the first sage is equal what? (Can you figure out the first sage's exact cards?)

- (11) The number a is positive and less than 1, and the number b is greater than 1. Which of the following numbers is greatest?

$$a \times b, \quad \frac{b}{a}, \quad \frac{a}{b}, \quad b, \quad b - a$$

- (12) Dustin chooses a 5-digit positive integer and deletes one of its digits to make a 4-digit number. The sum of this 4-digit number and the original 5-digit number is 52713. What is the sum of the digits of the original 5-digit number?

- (13) A certain boy always tells the truth on Thursdays and Fridays, always lies on Tuesdays, and randomly tells the truth or lies on other days of the week. On seven consecutive days he was asked what his name was, and on the first six days he gave the following answers in this order: Pax, Morgan, Pax, Morgan, Deven, Morgan. What did he answer on the seventh day?
- (14) The positive integers x, y, z satisfy $x \cdot y = 14$, $y \cdot z = 10$, and $x \cdot z = 35$. What is $x + y + z$?
- (15) The British mathematician August de Morgan claimed that he was x years old in the year of x^2 . He is known to have died in 1899. When was he born?
- (16) The product of my children's ages is equal to 1664. The oldest child is twice as old as the youngest. How many children do I have. (What are their ages?)

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- (17) When 999 was divided by a certain two digit number the remainder was 3. What is the remainder when 2001 is divided by this number?
- (18) A pair of integers is called *good* if their sum is equal to their product. How many good paris of integers are there?
- (19) What is the first digit of the smallest number in which the sum of the digits equals 2014?
- (20) A bottle of a volume of $\frac{1}{3}$ liter is filled $\frac{3}{4}$ with juice. How much juice will be left in the bottle after pouring out $\frac{1}{5}$ of a liter?

(21) What is the value of the expression:

$$(1^2 + 2^2 + 3^2 + \cdots + 2014^2) - (1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + 2013 \cdot 2015)?$$

(22) The sum of the smallest three-digit number whose digits add up to 8 and the largest three-digit number whose digits add up to 8 is?

(23) An island is inhabited by two types of people: truth-tellers and liars. The truth-tellers always speak the truth and the liars always lie. 25 of the island's inhabitants stood in a line. Each of them with the exception of the first person said: *The person directly in front of me is a liar*, while the person standing first in the line said: *Everyone standing behind me is a liar*. How many liars stood in the line?

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- (24) How many ten digit numbers can be constructed using the digits 1, 2 and 3 so that any two consecutive digits differ by exactly one?
- (25) Consider pairs of positive integers with sum no larger than 103 and quotient smaller than $\frac{1}{3}$. What is the largest possible quotient of any such pair?
- (26) Each of 18 cards is numbered with either a 4 or a 5. It turns out that the sum of all the numbers is divisible by 17. How many cards are labeled with a 4?
- (27) Derek, who is an avid fisherman, caught 12 fish over three consecutive days. On each day, he caught more fish than on the previous day. On the third day he caught fewer fish than the total number from the previous two days. How many fish did Derek catch on the third day?

(28) What is the angle formed by the hour and minute hand of a clock at 4 : 40 pm?

(29) How many solutions that can be expressed with positive integers does the equation below have?

$$a^2b + 14 = 2014$$

(30) If $\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdots \frac{a}{b} = 9$, then what is $a + b$?