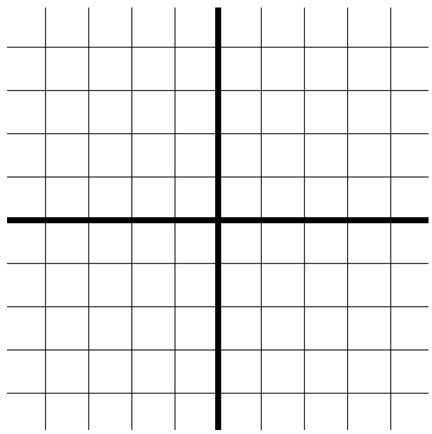
ABSOLUTE VALUE & DISTANCE

LAMC INTERMEDIATE GROUP - 2/02/14

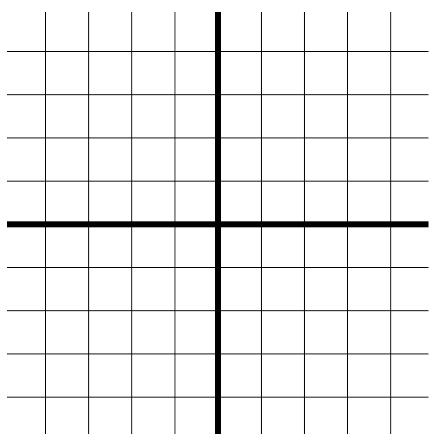
Warm Up

(1) Sketch the graphs of the following linear functions.

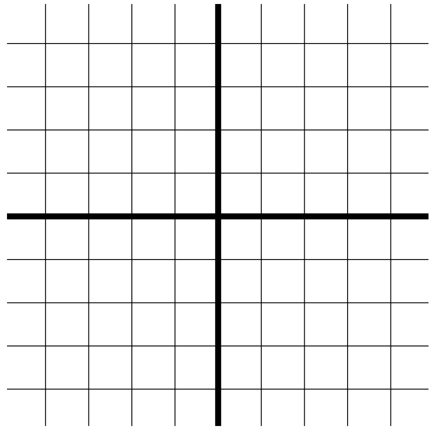
(a) $y = x$



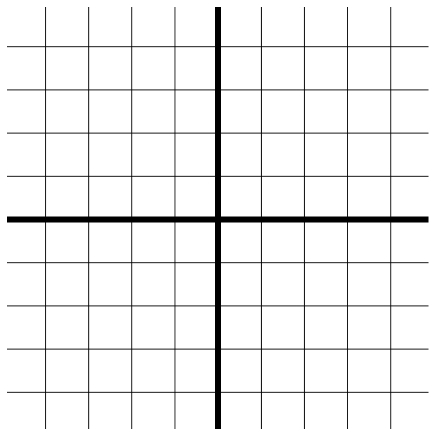
(b) $y = x + 5$



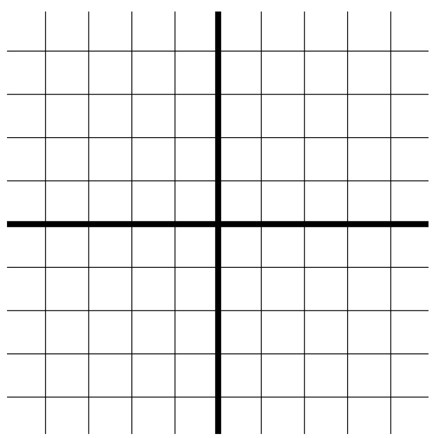
(c) $y = 2x$



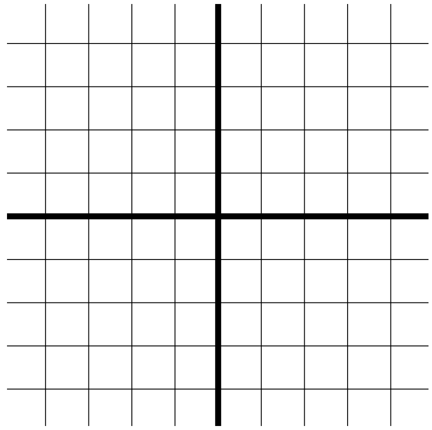
(d) $y = \frac{x}{2}$



(e) $y = -x$



(f) $y = -3x + 2$



Absolute value

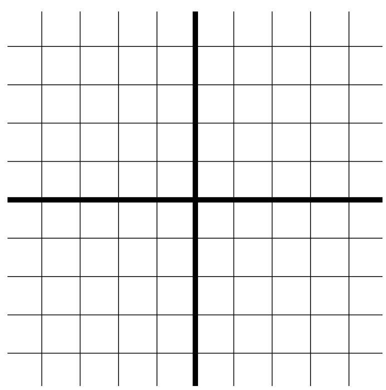
We define absolute value as follows:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$$

(We can also think of this as "distance to zero," an idea that will be made more precise later!)

(1) 0 is included in both cases of this piece-wise defined function. Is that a problem?

(2) Graph $y = |x|$.



(3) Let x be a fixed number. Is the sequence

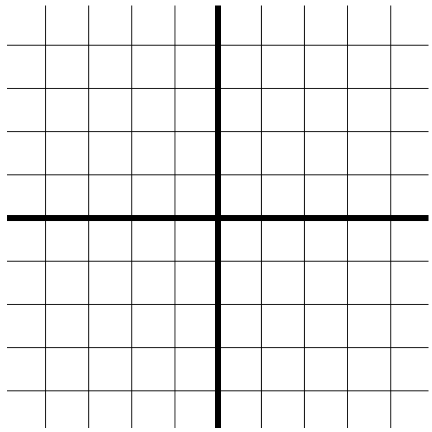
$$|x+1|, |x - \frac{1}{2}|, |x + \frac{1}{3}|, |x - \frac{1}{4}|, \cdots$$

convergent? If so, what does it converge to? Is there an x for which this sequence is monotone?

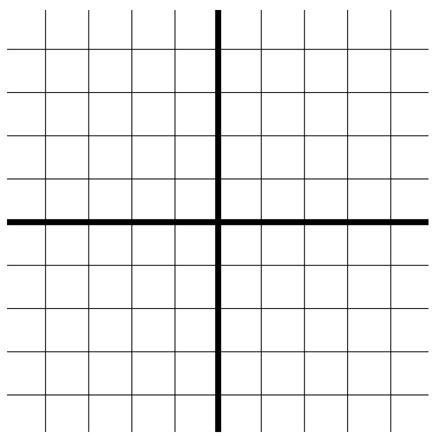
Graphs.

(1) Sketch the graphs of the following:

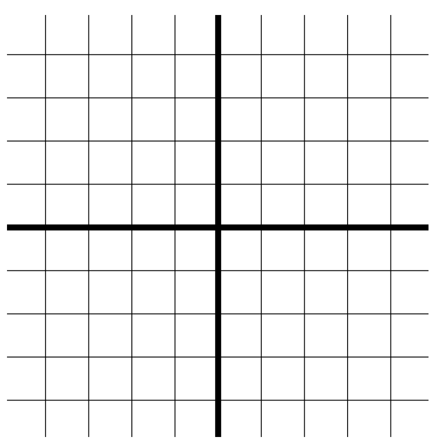
(a) $y = |x - 3|$



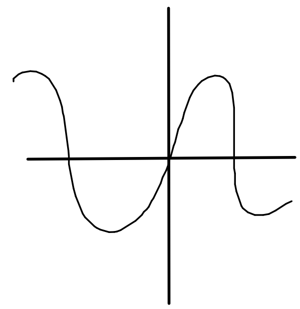
(b) $y = |x| - 3$



(c) $y = \big||x| - 3\big| - 3$



(2) Given the graph $y = f(x)$:



(a) Graph $y = |f(x)|$.

(b) Graph $y = f(|x|)$.

(c) Graph $y = |f(|x|)|$.

Equalities.

(1) For which x is $|x-2|=1$?

(2) For which x is $|x|-2=1$?

(3) For which x is $|x-3|=0$?

(4) For which x is $|x-2|-3=0$?

(5) For which x is $||x-3|-2|=1$?

Inequalities.

(1) For which x is $|x-2|<1$?

(2) For which x is $|x-3|>5$?

(3) For which x is $||x-1|-1|<6$?

Distances*

We call a function $d(x, y)$ a distance if it satisfies the following four distance relations:

- $d(x, y) \geq 0$ for all x and y.
- $d(x, x) = 0$ and $d(x, y) \neq 0$ when $x \neq y$.
- $d(x, y) = d(y, x)$ for all x and y.
- $d(x, z) \leq d(x, y) + d(x, z)$ for all $x, y,$ and z.

(1) Interpret these relations intuitively. For example, the first one says “Distance is always a non-negative number.” This makes sense because you shouldn’t be able to be -3 feet away from someone!

(2) In this problem we verify that the function $d(x, y) = |x - y|$ satisfies the first three relations.

(a) $|x - y| \geq 0$ for all x, y.

(b) $|x - x| = 0$ but $|x - y| > 0$ if $y \neq x$.

(c) $|y - x| = |x - y|$ for any x, y.

(3) **(Challenge!)** We would like to define a distance on the plane, (a distance between pairs (x_0, y_0) and (x_1, y_1)).

(a) One attempt to define an absolute value (distance to zero) is by

$$|(x_0, y_0)| = |x_0| + |y_0|.$$

Does the function

$$d((x_0, y_0), (x_1, y_1)) = |(x_0, y_0) - (x_1, y_1)|$$

satisfy the relations for a distance? Interpret this distance geometrically.

In the rest of this problem, we define another distance on the plane (using the Pythagorean theorem!).

(b) Show that $|x| = \sqrt{x^2}$. (**Hint:** Absolute value is defined piece-wise, so you'll have to check the equality in both cases!)

(c) Using the idea from (b) *[or (a) and the pythagorean theorem]*, come up with a suitable generalization of the absolute value to points in the plane (i.e. a distance to 0 in the plane).

(d) Verify that the first three conditions of a distance hold for your generalized absolute value (where the distance is defined as usual from an absolute value).

Homework

Solve the equations

(1)
$$|x+2|+|x+3|=1$$

(2)
$$|x-2|+|x-1|=1$$