

iii. a and b ($a, b \geq 0$)

(b) It turns out that the geometric mean of two nonnegative numbers will *always* be smaller than their arithmetic mean. **Prove it!**
(Hint: Find the difference between the arithmetic and geometric means for two arbitrary numbers a and b .)

(c) Find all pairs of nonnegative numbers such that their arithmetic and geometric means are equal.

4. Find the arithmetic and geometric means of the following sets of numbers:

(a) 1,2,3,6

(b) 0,4,8,20

(c) 4,4,4

5. Prove that $1 + x \geq 2\sqrt{x}$, if $x \geq 0$.

6. Prove that $x + \frac{1}{x} \geq 2$, if $x > 0$.

7. Prove that $\frac{x^2+y^2}{2} \geq xy$ for any x and y .

8. Prove that $2(x^2 + y^2) \geq (x + y)^2$ for any x and y .

9. The sum of two nonnegative numbers is 10. What are the maximum and minimum values of the sum of their squares?

10. Prove the AM-GM inequality for a set of four numbers.
(First, write an expression for the AM and the GM of the four numbers.)

11. Use mathematical induction to prove, for any natural number $n \geq 3$, that

$$\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} > \frac{3}{5}$$

12. (*) Use mathematical induction to prove the AM-GM inequality for a set of n numbers, where n is a power of 2.

13. Is it possible to pack the entire human population of the Earth in a cube with side length 2 miles? (Estimate relevant values; calculators ok)

- Review Problems -

14. There are 13 grey, 15 brown, and 17 red chameleons on Chromatic Island. When two chameleons of different colors meet, they both change their color to the third one. Is it possible that after some time, all the chameleons on the island are the same color?

15. Consider the following premise: You will get an extra credit point if you write a paper or if you solve the bonus test problem.
Your situation is as follows: Jeff didn't write a paper, but he got an extra credit point.
John asked, "Wow, how'd you solve that bonus test problem?!"
Jeff replied, "I didn't solve it."
Assign letters to each statement and form wffs to represent the situation. Is Jeff lying?