

**Wednesday, FEBRUARY 15, 2006**

57<sup>th</sup> Annual American Mathematics Contest 12

**AMC 12**



**Contest B**

**The MATHEMATICAL ASSOCIATION OF AMERICA  
American Mathematics Competitions**

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have 75 MINUTES to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

*Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 24<sup>th</sup> annual American Invitational Mathematics Examination (AIME) on Tuesday, March 7, 2006 or Wednesday, March 22, 2006. More details about the AIME and other information are on the back page of this test booklet.*

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The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 12 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, World Wide Web or media of any type is a violation of the competition rules.

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- What is  $(-1)^1 + (-1)^2 + \cdots + (-1)^{2006}$  ?  
(A)  $-2006$     (B)  $-1$     (C)  $0$     (D)  $1$     (E)  $2006$
- For real numbers  $x$  and  $y$ , define  $x \spadesuit y = (x + y)(x - y)$ . What is  $3 \spadesuit (4 \spadesuit 5)$  ?  
(A)  $-72$     (B)  $-27$     (C)  $-24$     (D)  $24$     (E)  $72$
- A football game was played between two teams, the Cougars and the Panthers. The two teams scored a total of 34 points, and the Cougars won by a margin of 14 points. How many points did the Panthers score?  
(A) 10    (B) 14    (C) 17    (D) 20    (E) 24
- Mary is about to pay for five items at the grocery store. The prices of the items are \$7.99, \$4.99, \$2.99, \$1.99, and \$0.99. Mary will pay with a twenty-dollar bill. Which of the following is closest to the percentage of the \$20.00 that she will receive in change?  
(A) 5    (B) 10    (C) 15    (D) 20    (E) 25
- John is walking east at a speed of 3 miles per hour, while Bob is also walking east, but at a speed of 5 miles per hour. If Bob is now 1 mile west of John, how many minutes will it take for Bob to catch up to John?  
(A) 30    (B) 50    (C) 60    (D) 90    (E) 120
- Francesca uses 100 grams of lemon juice, 100 grams of sugar, and 400 grams of water to make lemonade. There are 25 calories in 100 grams of lemon juice and 386 calories in 100 grams of sugar. Water contains no calories. How many calories are in 200 grams of her lemonade?  
(A) 129    (B) 137    (C) 174    (D) 223    (E) 411
- Mr. and Mrs. Lopez have two children. When they get into their family car, two people sit in the front, and the other two sit in the back. Either Mr. Lopez or Mrs. Lopez must sit in the driver's seat. How many seating arrangements are possible?  
(A) 4    (B) 12    (C) 16    (D) 24    (E) 48
- The lines  $x = \frac{1}{4}y + a$  and  $y = \frac{1}{4}x + b$  intersect at the point  $(1, 2)$ . What is  $a + b$  ?  
(A) 0    (B)  $\frac{3}{4}$     (C) 1    (D) 2    (E)  $\frac{9}{4}$
- How many even three-digit integers have the property that their digits, read left to right, are in strictly increasing order?  
(A) 21    (B) 34    (C) 51    (D) 72    (E) 150

10. In a triangle with integer side lengths, one side is three times as long as a second side, and the length of the third side is 15. What is the greatest possible perimeter of the triangle?

(A) 43      (B) 44      (C) 45      (D) 46      (E) 47

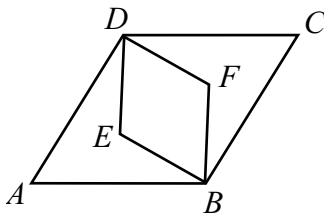
11. Joe and JoAnn each bought 12 ounces of coffee in a 16-ounce cup. Joe drank 2 ounces of his coffee and then added 2 ounces of cream. JoAnn added 2 ounces of cream, stirred the coffee well, and then drank 2 ounces. What is the resulting ratio of the amount of cream in Joe's coffee to that in JoAnn's coffee?

(A)  $\frac{6}{7}$       (B)  $\frac{13}{14}$       (C) 1      (D)  $\frac{14}{13}$       (E)  $\frac{7}{6}$

12. The parabola  $y = ax^2 + bx + c$  has vertex  $(p, p)$  and  $y$ -intercept  $(0, -p)$ , where  $p \neq 0$ . What is  $b$ ?

(A)  $-p$       (B) 0      (C) 2      (D) 4      (E)  $p$

13. Rhombus  $ABCD$  is similar to rhombus  $BFDE$ . The area of rhombus  $ABCD$  is 24, and  $\angle BAD = 60^\circ$ . What is the area of rhombus  $BFDE$ ?

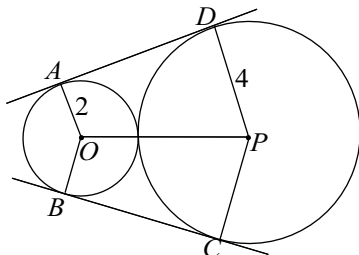


(A) 6      (B)  $4\sqrt{3}$       (C) 8      (D) 9      (E)  $6\sqrt{3}$

14. Elmo makes  $N$  sandwiches for a fundraiser. For each sandwich he uses  $B$  globs of peanut butter at 4¢ per glob and  $J$  blobs of jam at 5¢ per blob. The cost of the peanut butter and jam to make all the sandwiches is \$2.53. Assume that  $B$ ,  $J$ , and  $N$  are positive integers with  $N > 1$ . What is the cost of the jam Elmo uses to make the sandwiches?

(A) \$1.05      (B) \$1.25      (C) \$1.45      (D) \$1.65      (E) \$1.85

15. Circles with centers  $O$  and  $P$  have radii 2 and 4, respectively, and are externally tangent. Points  $A$  and  $B$  are on the circle centered at  $O$ , and points  $C$  and  $D$  are on the circle centered at  $P$ , such that  $\overline{AD}$  and  $\overline{BC}$  are common external tangents to the circles. What is the area of hexagon  $AOBCPD$ ?



- (A)  $18\sqrt{3}$     (B)  $24\sqrt{2}$     (C) 36    (D)  $24\sqrt{3}$     (E)  $32\sqrt{2}$
16. Regular hexagon  $ABCDEF$  has vertices  $A$  and  $C$  at  $(0,0)$  and  $(7,1)$ , respectively. What is its area?
- (A)  $20\sqrt{3}$     (B)  $22\sqrt{3}$     (C)  $25\sqrt{3}$     (D)  $27\sqrt{3}$     (E) 50
17. For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5, and 6 on each die are in the ratio  $1 : 2 : 3 : 4 : 5 : 6$ . What is the probability of rolling a total of 7 on the two dice?
- (A)  $\frac{4}{63}$     (B)  $\frac{1}{8}$     (C)  $\frac{8}{63}$     (D)  $\frac{1}{6}$     (E)  $\frac{2}{7}$
18. An object in the plane moves from one lattice point to another. At each step, the object may move one unit to the right, one unit to the left, one unit up, or one unit down. If the object starts at the origin and takes a ten-step path, how many different points could be the final point?
- (A) 120    (B) 121    (C) 221    (D) 230    (E) 231
19. Mr. Jones has eight children of different ages. On a family trip his oldest child, who is 9, spots a license plate with a 4-digit number in which each of two digits appears two times. "Look, daddy!" she exclaims. "That number is evenly divisible by the age of each of us kids!" "That's right," replies Mr. Jones, "and the last two digits just happen to be my age." Which of the following is *not* the age of one of Mr. Jones's children?
- (A) 4    (B) 5    (C) 6    (D) 7    (E) 8
20. Let  $x$  be chosen at random from the interval  $(0,1)$ . What is the probability that

$$\lfloor \log_{10} 4x \rfloor - \lfloor \log_{10} x \rfloor = 0?$$

Here  $\lfloor x \rfloor$  denotes the greatest integer that is less than or equal to  $x$ .

- (A)  $\frac{1}{8}$     (B)  $\frac{3}{20}$     (C)  $\frac{1}{6}$     (D)  $\frac{1}{5}$     (E)  $\frac{1}{4}$

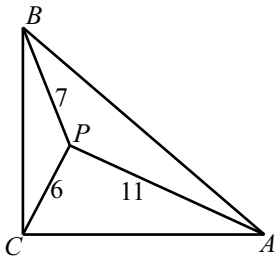
21. Rectangle  $ABCD$  has area  $2006$ . An ellipse with area  $2006\pi$  passes through  $A$  and  $C$  and has foci at  $B$  and  $D$ . What is the perimeter of the rectangle? (The area of an ellipse is  $\pi ab$ , where  $2a$  and  $2b$  are the lengths of its axes.)

(A)  $\frac{16\sqrt{2006}}{\pi}$     (B)  $\frac{1003}{4}$     (C)  $8\sqrt{1003}$     (D)  $6\sqrt{2006}$     (E)  $\frac{32\sqrt{1003}}{\pi}$

22. Suppose  $a, b$ , and  $c$  are positive integers with  $a+b+c = 2006$ , and  $a!b!c! = m \cdot 10^n$ , where  $m$  and  $n$  are integers and  $m$  is not divisible by  $10$ . What is the smallest possible value of  $n$ ?

(A) 489    (B) 492    (C) 495    (D) 498    (E) 501

23. Isosceles  $\triangle ABC$  has a right angle at  $C$ . Point  $P$  is inside  $\triangle ABC$ , such that  $PA = 11$ ,  $PB = 7$ , and  $PC = 6$ . Legs  $\overline{AC}$  and  $\overline{BC}$  have length  $s = \sqrt{a + b\sqrt{2}}$ , where  $a$  and  $b$  are positive integers. What is  $a + b$ ?



(A) 85    (B) 91    (C) 108    (D) 121    (E) 127

24. Let  $S$  be the set of all points  $(x, y)$  in the coordinate plane such that  $0 \leq x \leq \frac{\pi}{2}$  and  $0 \leq y \leq \frac{\pi}{2}$ . What is the area of the subset of  $S$  for which

$$\sin^2 x - \sin x \sin y + \sin^2 y \leq \frac{3}{4}?$$

(A)  $\frac{\pi^2}{9}$     (B)  $\frac{\pi^2}{8}$     (C)  $\frac{\pi^2}{6}$     (D)  $\frac{3\pi^2}{16}$     (E)  $\frac{2\pi^2}{9}$

25. A sequence  $a_1, a_2, \dots$  of non-negative integers is defined by the rule  $a_{n+2} = |a_{n+1} - a_n|$  for  $n \geq 1$ . If  $a_1 = 999$ ,  $a_2 < 999$ , and  $a_{2006} = 1$ , how many different values of  $a_2$  are possible?

(A) 165    (B) 324    (C) 495    (D) 499    (E) 660

## WRITE TO US!

*Correspondence about the problems and solutions for this AMC 12 and orders for any of the publications listed below should be addressed to:*

American Mathematics Competitions  
University of Nebraska, P.O. Box 81606  
Lincoln, NE 68501-1606  
Phone: 402-472-2257; Fax: 402-472-6087; email: amcinfo@unl.edu

*The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:*

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## 2006 AIME

The AIME will be held on Tuesday, March 7, 2006 with the alternate on Wednesday, March 22, 2006. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above, or finish in the top 1% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on April 18 and 19, 2006. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

## **PUBLICATIONS**

A complete listing of current publications, with ordering instructions, is at our web site:

2006

AMC 12 – Contest B

# DO NOT OPEN UNTIL WEDNESDAY, February 15, 2006

**\*\*Administration On An Earlier Date Will Disqualify  
Your School's Results\*\***

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 15.** Nothing is needed from inside this package until February 15.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC no later than 24 hours following the examination.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, World Wide Web or media of any type is a violation of the competition rules.*

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