

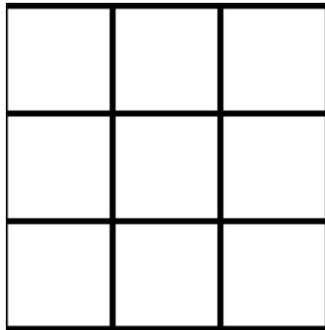
## Fun and Games on a Chess Board II

Early Elementary

January 27, 2014

Last week we counted the number of squares of size  $2 \times 2$  on a chessboard. Today, let's start by counting the number of squares of size  $3 \times 3$  on a chessboard.

Idea: *Instead of counting  $3 \times 3$  squares, we will count the small  $1 \times 1$  squares which can serve as the **left lower** corners of the  $3 \times 3$  squares that fit on the chessboard.*



First, shade the left lower corner of the  $3 \times 3$  square above.

1. For each of the squares below, decide if it can be a left lower corner of a  $3 \times 3$  square:

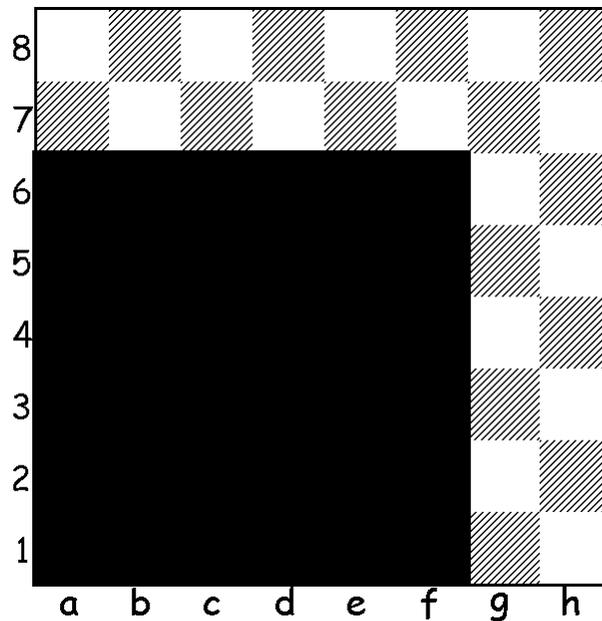
(a) square e6                      Yes

(b) square g3                      No

(c) square a7                      No

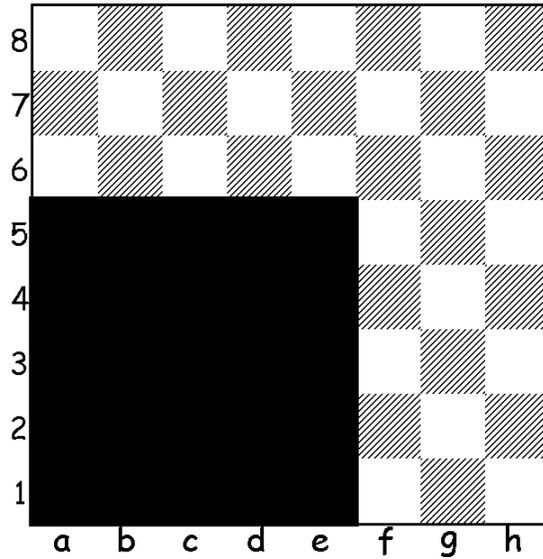
(d) square f6                      Yes

Now color *all*  $1 \times 1$  squares that can serve as the left lower corners of a  $3 \times 3$  square:

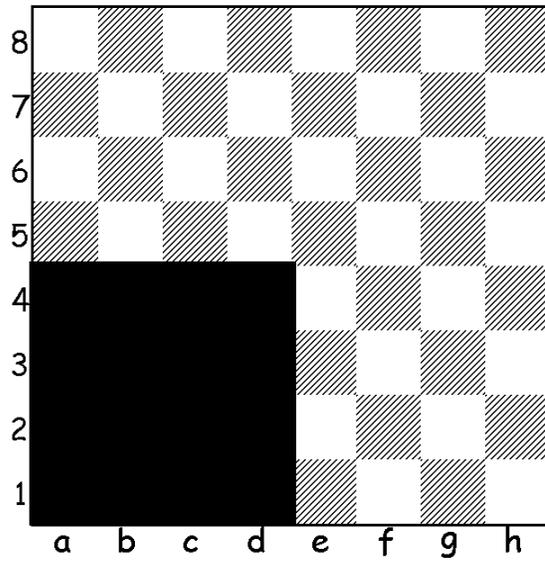


How many  $3 \times 3$  squares can you fit onto a chessboard?

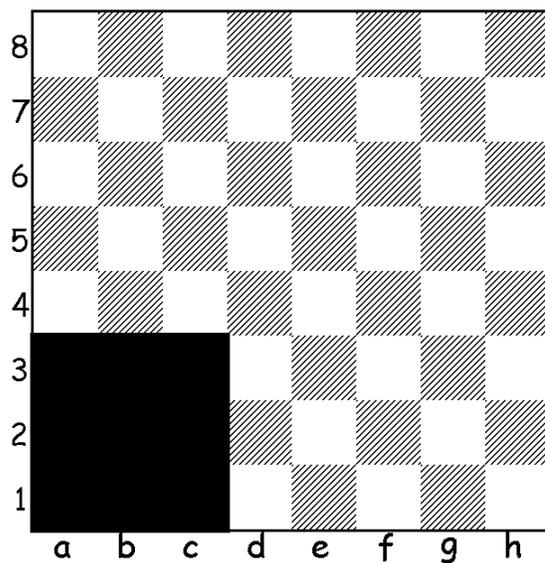
Homework: Color *all*  $1 \times 1$  squares that can serve as the left lower corners of a  $4 \times 4$  square:



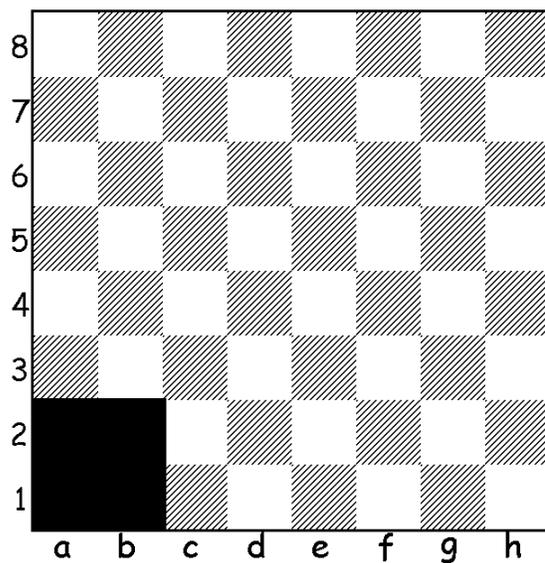
Color all  $1 \times 1$  squares that can serve as the left lower corners of a  $5 \times 5$  square:



Color all  $1 \times 1$  squares that can serve as the left lower corners of a  $6 \times 6$  square:



Color all  $1 \times 1$  squares that can serve as the left lower corners of a  $7 \times 7$  square:



Fill out the table below with the numbers of squares:

| size of the square | # of squares of this size |
|--------------------|---------------------------|
| $1 \times 1$       | 64                        |
| $2 \times 2$       | 49                        |
| $3 \times 3$       | 36                        |
| $4 \times 4$       | 25                        |
| $5 \times 5$       | 16                        |
| $6 \times 6$       | 9                         |
| $7 \times 7$       | 4                         |
| $8 \times 8$       | 1                         |

## II *Rook Race Game*

Two players are playing the following game:

- **Initial position:** Two rooks are placed on two squares of a chessboard.
- **Move:** Move *any* of the rooks to the right by any number of squares.
- **Goal:** To be the *last* person to reach the rightmost square.

1. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. Only one of the players (first or second) has a winning strategy. You need to find it. Here are the initial positions:

(a) Rook I on f3,      Rook II on f6  
Which player can win? ( **Player II** )

(b) Rook I on d3,      Rook II on d6  
Which player can win? ( **Player II** )

(c) In general, if both Rooks are the same number of squares away from the right edge, which player can always win? How

**Solution:** The second player will win. On his first move, player I is forced to move one of the rooks ahead of the other one. As long as Player II makes sure to line the

two rooks up again, this strategy will insure that Player I will reach the right edge first.

2. Now use the following initial positions:

(a) Rook I on f3,      Rook II on d6  
Which player can win? (**Player I**)

(b) Rook I on e3,      Rook II on a6  
Which player can win? (**Player I**)

(c) Rook I on a3,      Rook II on b6  
Which player can win? (**Player I**)

(d) Rook I on c3,      Rook II on g6  
Which player can win? (**Player I**)

(e) In general, if the rooks are a different number of squares away from the right edge, which player can always win? How?

**Player I can always win, by lining up the two rooks. Once they are lined up, no matter what move Player II makes, Player I can line the rooks back up, thus guaranteeing that he will reach the edge of the board second.**

## II *Put Rook Into the Corner Game*

Two players are playing the following game:

- **Initial position:** One Rook is placed somewhere on the chessboard.
  - **Move:** Move the Rook down or left by any number of squares.
  - **Goal:** To put the Rook into the left lower corner.
3. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. In every position, only one of the players (first or second) has a winning strategy. You need to find it. Here are the initial positions:
- (a) Rook on c3. Which player can win?  
(**Player II**)
  - (b) Rook on d4. Which player can win?  
(**Player II**)
  - (c) Rook on f7. Which player can win?  
(**Player I**)
  - (d) Rook on g8. Which player can win?  
(**Player I**)

- (e) In general, if the Rook is on the diagonal connecting squares a1 and h8, which player can win? How?

**Player II will win in this case. Player I is forced to move the rook away from the diagonal, thus allowing Player II to always bring the rook back to the diagonal. Since the corner piece is on the diagonal, this ensures that Player II will win.**

- (f) How does the game change if the Rook is placed away from the diagonal? Which player can win now?

**Player I can now always win, by using his first move to take the rook to the diagonal. Once it is on the diagonal, the second player must move it away from the diagonal, ensuring that Player I can always bring the rook back to the diagonal, and ultimately win.**

### **Homework:**

1. Play both the *Rook Race* and *Put Rook into the corner* games at home with your parents, friends, brothers and siblings.