

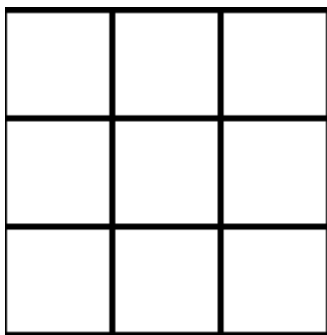
Fun and Games on a Chess Board II

Early Elementary

January 26, 2014

Last week we counted the number of squares of size 2×2 on a chessboard. Today, lets start by counting the number of squares of size 3×3 on a chessboard.

Idea: *Instead of counting 3×3 squares, we will count the small 1×1 squares which can serve as the **left lower** corners of the 3×3 squares that fit on the chessboard.*



First, shade the left lower corner of the 3×3 square above.

1. For each of the squares below, decide if it can be a left lower corner of a 3×3 square:

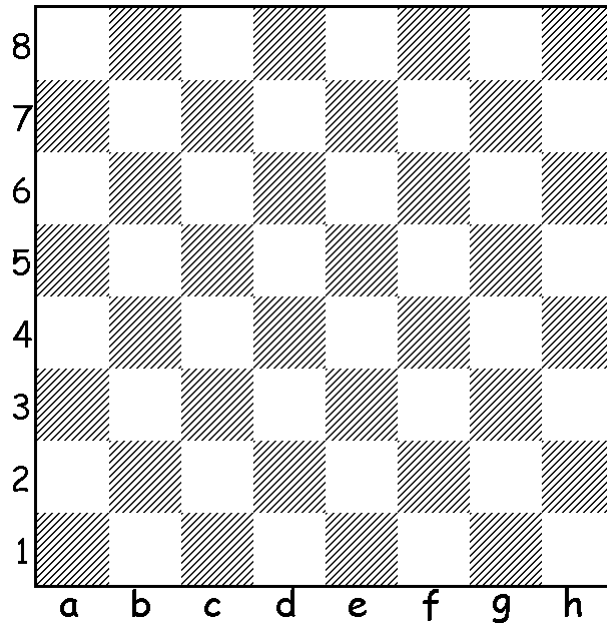
(a) square e6 Yes No

(b) square g3 Yes No

(c) square a7 Yes No

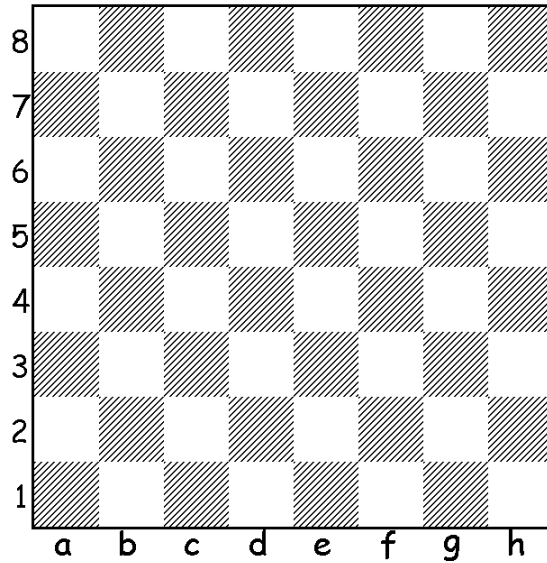
(d) square f6 Yes No

Now color *all* 1×1 squares that can serve as the left lower corners of a 3×3 square:

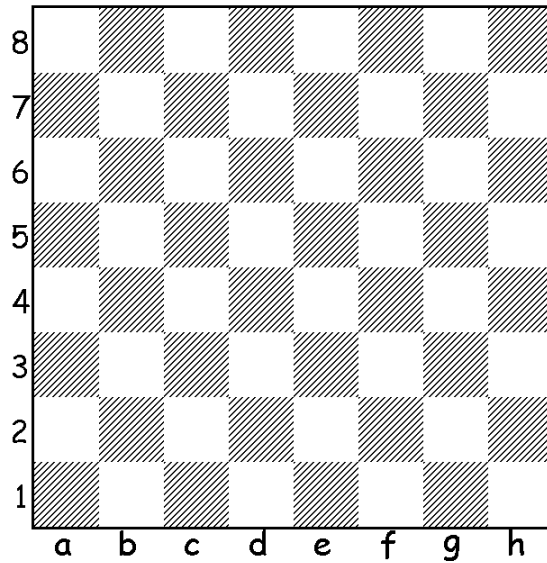


How many 3×3 squares can you fit onto a chessboard?

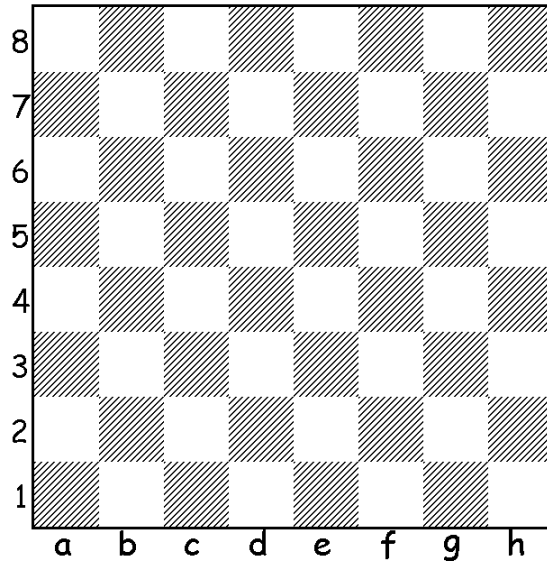
Homework: Color *all* 1×1 squares that can serve as the left lower corners of a 4×4 square:



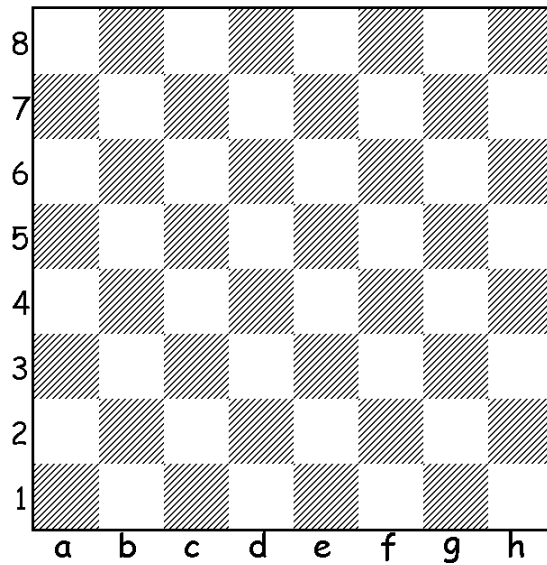
Color all 1×1 squares that can serve as the left lower corners of a 5×5 square:



Color all 1×1 squares that can serve as the left lower corners of a 6×6 square:



Color all 1×1 squares that can serve as the left lower corners of a 7×7 square:



Fill out the table below with the numbers of squares:

size of the square	# of squares of this size
1×1	
2×2	
3×3	
4×4	
5×5	
6×6	
7×7	
8×8	

II *Rook Race Game*

Two players are playing the following game:

- **Initial position:** Two rooks are placed on two squares of a chessboard.
- **Move:** Move *any* of the rooks to the right by any number of squares.
- **Goal:** To be the *last* person to reach the rightmost square.

1. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. Only one of the players (first or second) has a winning strategy. You need to find it. Here are the initial positions:

(a) Rook I on f3, Rook II on f6
Which player can win? (Player I or Player II)

(b) Rook I on d3, Rook II on d6
Which player can win? (Player I or Player II)

(c) In general, if both Rooks are the same number of squares away from the right edge, which player can always win? How?

2. Now use the following initial positions:

- (a) Rook I on f3, Rook II on d6
Which player can win? (Player I or Player II)

- (b) Rook I on e3, Rook II on a6
Which player can win? (Player I or Player II)

- (c) Rook I on a3, Rook II on b6
Which player can win? (Player I or Player II)

- (d) Rook I on c3, Rook II on g6
Which player can win? (Player I or Player II)

- (e) In general, if the rooks are a different number of squares away from the right edge, which player can always win? How?

- (f) If you think you can handle any Rook race game, please challenge one of the instructors to play with you. The instructor will set up an initial position, and you will have a choice of being Player I or Player II.

GOOD LUCK!

II *Put Rook Into the Corner Game*

Two players are playing the following game:

- **Initial position:** One Rook is placed somewhere on the chessboard.
 - **Move:** Move the Rook down or left by any number of squares.
 - **Goal:** To put the Rook into the left lower corner.
3. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. In every position, only one of the players (first or second) has a winning strategy. You need to find it. Here are the initial positions:
- (a) Rook on c3. Which player can win?
(Player I or Player II)
 - (b) Rook on d4. Which player can win?
(Player I or Player II)
 - (c) Rook on f7. Which player can win?
(Player I or Player II)
 - (d) Rook on g8. Which player can win?
(Player I or Player II)

- (e) In general, if the Rook is on the diagonal connecting squares a1 and h8, which player can win? How?
- (f) How does the game change if the Rook is placed away from the diagonal? Which player can win now?

Homework:

1. Play both the *Rook Race* and *Put Rook into the corner* games at home with your parents, friends, brothers and siblings.