

Fun and Games on a Chess Board

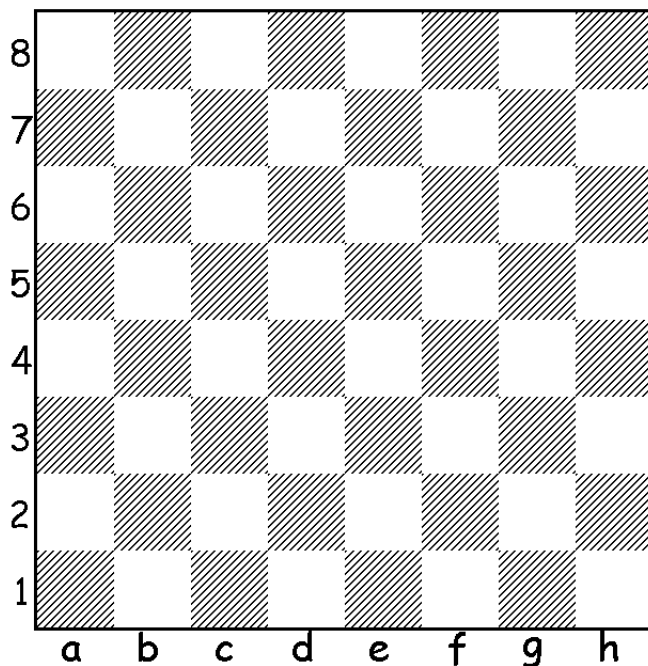
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I Names of squares on the chess board

Color the following squares on the chessboard below:

c3, c4, c5, c6, d5, e4, f3, f4, f5, f6



What letter do these squares form together?

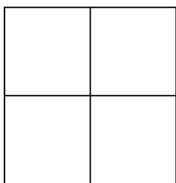
II How many squares are there on a chessboard?

A chessboard itself is a square with side 8.

1. The number of 1×1 squares on the chess board is .

2. What about bigger squares?

Let's first count squares of size 2×2 :



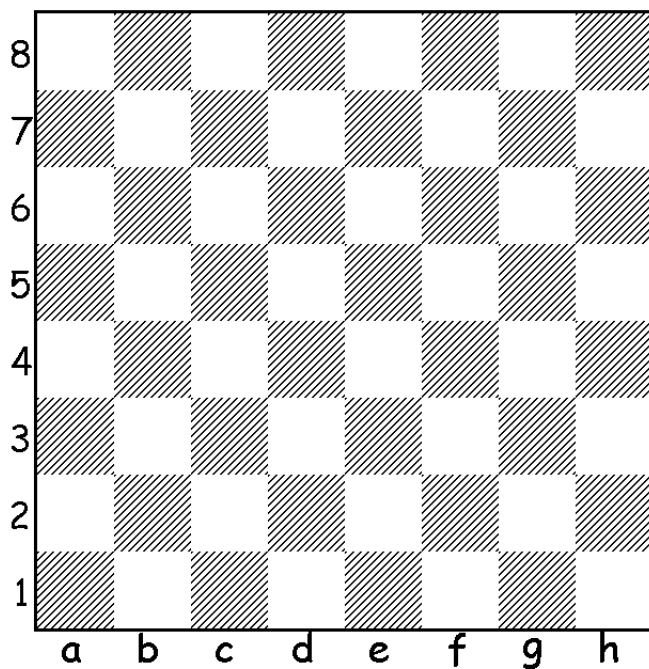
Idea: *Instead of counting 2×2 squares, we will count the small 1×1 squares which can serve as the left lower corners of the 2×2 squares that fit on the chessboard.*

First, shade the left lower corner of the 2×2 square above.

For each of the squares below, decide if it can be a left lower corner of a 2×2 square:

- | | | |
|---------------|-----|----|
| (a) square c3 | Yes | No |
| (b) square g6 | Yes | No |
| (c) square f8 | Yes | No |
| (d) square h2 | Yes | No |

Now color *all* 1×1 squares that can serve as the left lower corners of a 2×2 square:



How many 2×2 squares can you fit onto a chessboard?

3. For each of the squares below, decide if it can be a left lower corner of a 3×3 square:

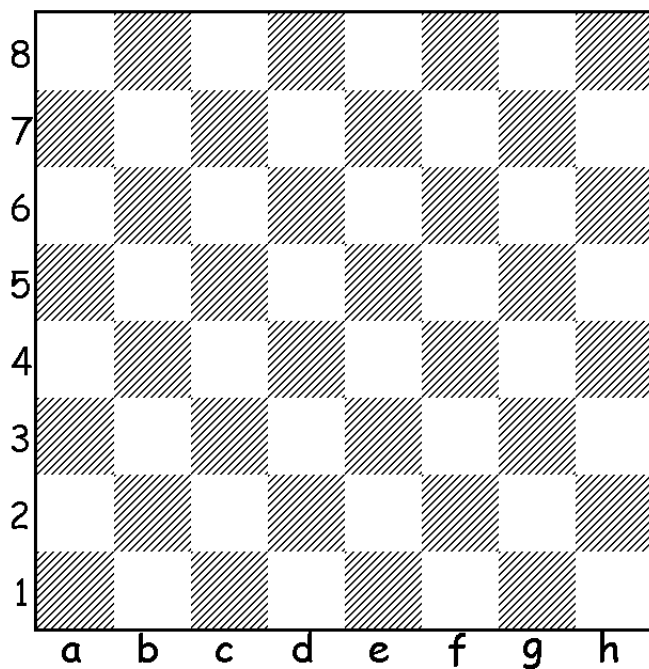
(a) square e6 Yes No

(b) square g3 Yes No

(c) square a7 Yes No



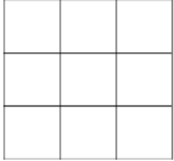
(d) square f6 Yes No

Now color *all* 1×1 squares that can serve as the left lower corners of a 3×3 square:



How many 3×3 squares can you fit onto a chessboard?

Now you can fill out the table below:

Type of Square	Number of such squares
	
	
	

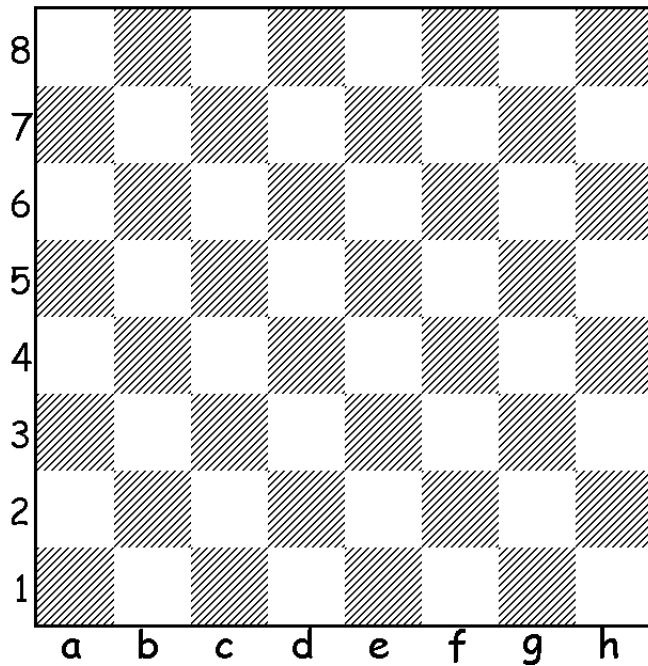
Homework

Count the number of 4×4 , 5×5 , 6×6 and 7×7 squares on the chessboard in the same way. In each case, use a chessboard picture (on the next pages) to shade all the 1×1 squares that can be left lower corners of the bigger squares that fit completely onto the chessboard.

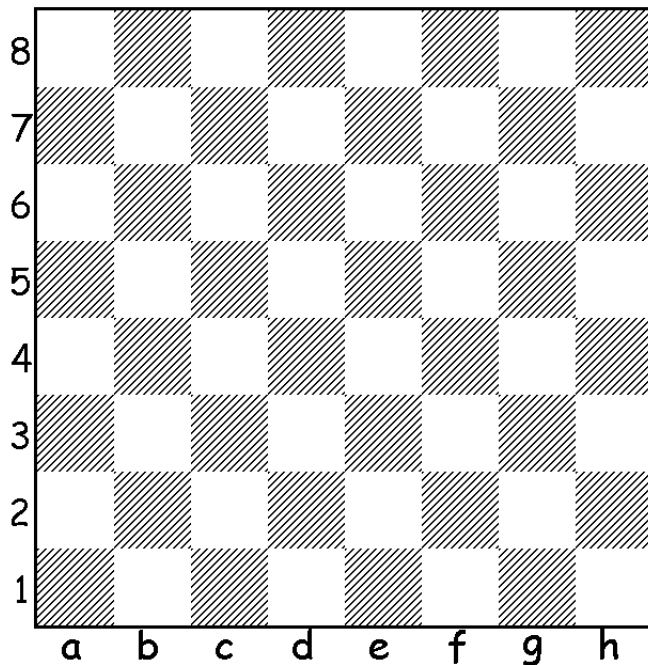
Then, fill out the table

size of the square	# of squares of this size
1×1	
2×2	
3×3	
4×4	
5×5	
6×6	
7×7	
8×8	

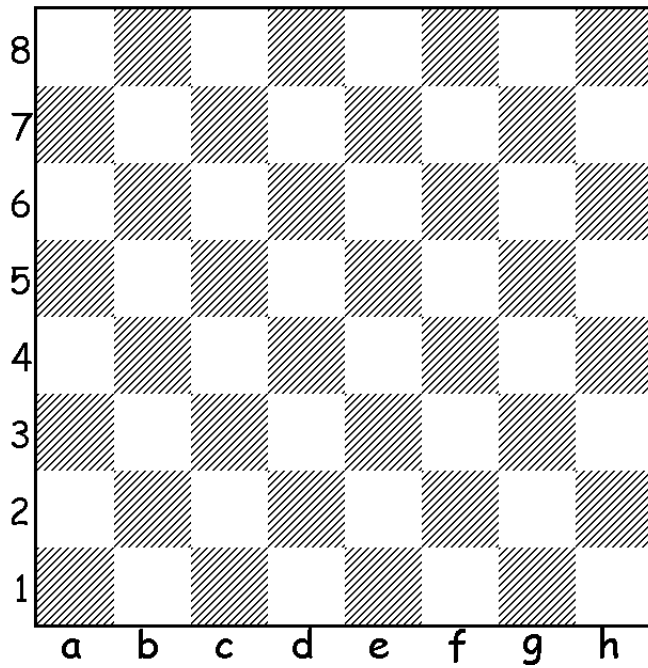
Now color *all* 1×1 squares that can serve as the left lower corners of a 4×4 square:



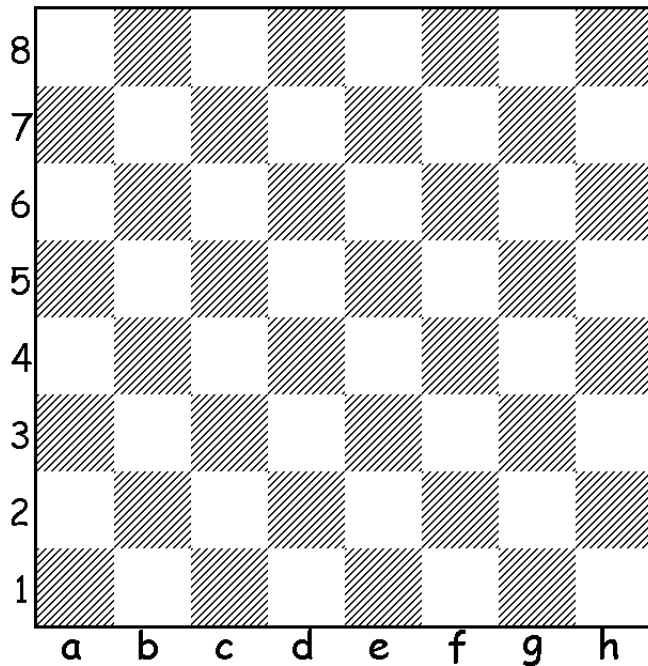
Color all 1×1 squares that can serve as the left lower corners of a 5×5 square:



Color all 1×1 squares that can serve as the left lower corners of a 6×6 square:



Color all 1×1 squares that can serve as the left lower corners of a 7×7 square:



II *Rook Race Game*

Two players are playing the following game:

- **Initial position:** Two rooks are placed on two squares of a chessboard.
 - **Move:** Move *any* of the rooks to the right by any number of squares.
 - **Goal:** To be the *last* person to reach the rightmost square.
1. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. Only one of the players (first or second) has a winning strategy. You need to find it.

Here are the initial positions:

- (a) Rook I on f3, Rook II on f6
Which player can win? (Player I or Player II)
- (b) Rook I on d3, Rook II on d6
Which player can win? (Player I or Player II)
- (c) In general, if both Rooks are the same number of squares away from the right edge, which player can win? How?

2. Now use the following initial positions:

- (a) Rook I on f3, Rook II on d6
Which player can win? (Player I or Player II)

- (b) Rook I on e3, Rook II on a6
Which player can win? (Player I or Player II)

- (c) Rook I on a3, Rook II on b6
Which player can win? (Player I or Player II)

- (d) Rook I on c3, Rook II on g6
Which player can win? (Player I or Player II)

- (e) In general, if the rooks are different number of squares away from the right edge, which player can win? How?

- (f) If you think you can handle any Rook race game, please challenge an instructor to play with you. The instructor will set up an initial position, and you will have a choice of being Player I or Player II.

GOOD LUCK!

II *Put Rook Into the Corner Game*

Two players are playing the following game:

- **Initial position:** One Rook is placed somewhere a chessboard.
 - **Move:** Move the Rook down or left by any number of squares.
 - **Goal:** To put the Rook into the left lower corner.
3. Play this game with your partner several times. Try to come up with a winning strategy. That is, come up with a method that allows you to win no matter what your opponent does. In every position, only one of the players (first or second) has a winning strategy. You need to find it.

Here are the initial positions:

(a) Rook c3

Which player can win? (Player I or Player II)

(b) Rook on d4, Rook II on d6

Which player can win? (Player I or Player II)

(c) Rook on d4, Rook II on f7

Which player can win? (Player I or Player II)

(d) In general, if the Rook is on the diagonal connecting squares a1 and h8, which player can win? How?

- (e) How does the game change if the Rook is placed away from the diagonal? Which player can win now?

Homework: Play both the *Rook Race* and *Put Rook into the corner* games at home with your parents, friends, brothers and sisters. Can you explain how these two games are alike?