

Fibonacci Sequence

January 19, 2014

Today we will learn about the *Fibonacci sequence*, one of the most famous sequences of numbers in mathematics. To construct it, we will look at the pattern of growth of the number of rabbits assuming we have one pair in the beginning and follow the rules below.

Rules:

1. Start with a pair of *new rabbits*.
2. When the rabbits are one month old, they become *teenagers*.
3. When the rabbits are two months old, they become *adults*.
4. A pair of *adult* rabbits gives birth to a pair of *newborn* rabbits every month.
5. Once a rabbit becomes an adult, it remains an adult forever. Rabbits never die.

1. If a pair of rabbits is born in December, which month will their first pair of children be born in?

2. Suppose there is 1 pair of newborn rabbits in December (Month 0).
 - (a) How many pairs of each type of rabbit (newborn, teenager, and adult) will there be in Month 1?
 - i. Adults:

 - ii. Teenagers:

 - iii. Newborns:

 - (b) What about Month 2?
 - i. Adults:

 - ii. Teenagers:

 - iii. Newborns:

3. Suppose there are 5 pairs of newborn rabbits in a given month.
 - (a) How many pairs of adults must there be in that same month?

 - (b) How many pairs of teenagers will there be in the next month?

4. Suppose there are 3 pairs of newborn rabbits, 2 pair of teenage rabbits, and 3 pairs of adult rabbits in a given month.

(a) How many pairs of each type of rabbit will there be in the next month?

i. Adults:

ii. Teenagers:

iii. Newborns:

(b) What about the next month?

i. Adults:

ii. Teenagers:

iii. Newborns:

5. Starting with one pair of newborn rabbits in Month 0, use the rules to fill in the chart below:

Month	0	1	2	3	4	5	6	7	8	9
Newborn	1									
Teenager	0									
Adult	0									
Total	1									

6. The sequence of numbers that represent the total number of pairs of rabbits is known as the Fibonacci sequence. Let F_n be the total number of pairs of rabbits in month n .

(a) Find the following Fibonacci numbers:

- $F_3 =$

- $F_5 =$

- $F_8 =$

(b) How would you denote the number of pairs of rabbits in the month just before n ?

(c) How would you denote the number of pairs of rabbits in the month before that?

(d) How is F_2 related to F_0 and F_1 ?

(e) How is F_5 related to F_3 and F_4 ?

(f) How is the total number of pairs in month n related to the total number of pairs in the previous two months?

7. We are going to look at how the number of pairs of rabbits change through a period of 3 months. Draw arrows representing each of the following types of events:

- (a) Every adult pair of rabbits continues to be adults in the next month (use *red*);
- (b) Every teenage rabbit becomes an adult next month (use *blue*);
- (c) Every pair of adult rabbits gives birth to a pair of newborn rabbits (use *green* curved arrow¹);
- (d) Every pair of newborns becomes a pair of teenagers in the next month (use *blue*);

	2 months ago		last month		current month	
Newborns						
Teenagers						
Adults						

¹This arrow should go up from adults to newborns in the same month, but should not go through the dot representing teenagers

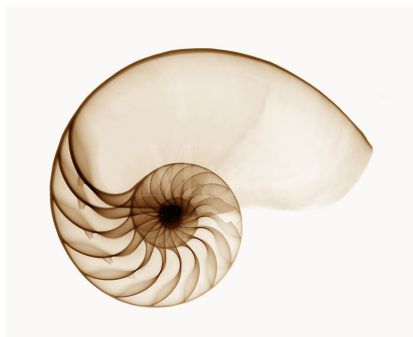
8. We noticed above that any number in the Fibonacci Sequence is equal to the sum of the previous two numbers. Let's figure out how the table with arrows proves this.

(a) Adult and teenage rabbits are considered to be *old* rabbits, while newborn rabbits are *new* rabbits. Looking at table above, determine how the number of pairs of old rabbits in the current month and the total number of pairs of rabbits from last month are related?

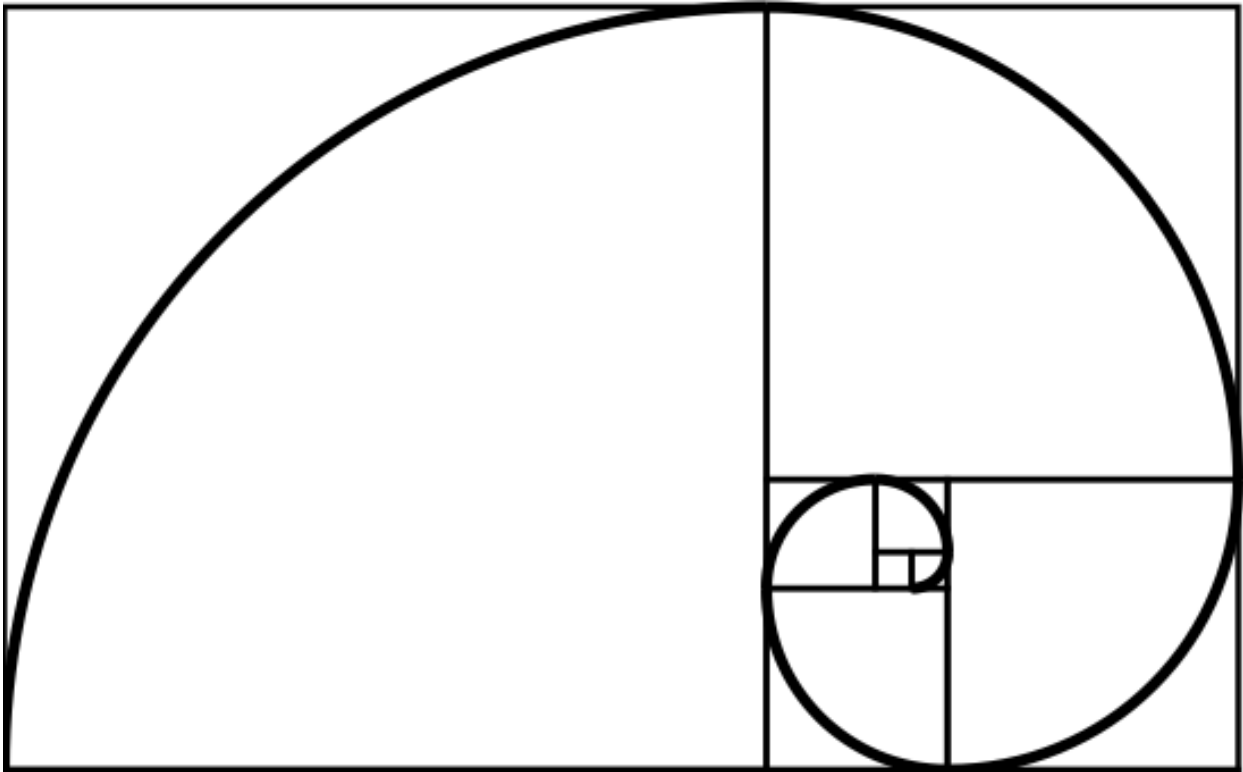
(b) How are the number of pairs of new rabbits in the current month and the total number of pairs of rabbits 2 months ago related?

(c) Adding up the number of pairs of old rabbits and new rabbits in the current month gives the total number of pairs. Using your answers from parts (a) and (b), show that the total number of pairs in the current month equals to the sum of the pairs from the previous 2 months.

9. Fibonacci sequence is also seen in other instances of nature! Here is the spiral of a Nautical Seashell:



Here is a diagram showing how the seashell is constructed. Assume that the side lengths of the smallest, inside squares are 1 cm. Find the side lengths of the rest of the squares by starting from the center and working outward (following the spiral of the shell).



What do you notice about the side lengths of the squares?

10. Create a pattern similar to Fibonacci Sequence starting with 2 pairs of newborn rabbits in month 0. Write down the first 8 numbers in the sequence.
11. Assume the first two numbers in a sequence are 1 and 3. Using same rule as in the Fibonacci sequence, write down the first 8 numbers in the new sequence.