

1. No. There are 64 squares on the chessboard, which is not divisible by 3. Thinking in terms of invariants, consider the process of adding 3×1 tiles. The quantity $I = \text{the remainder of the division: (number of squares)/3}$ is an invariant, because it is unchanged by this process. Since we start out with $I = 0$, we cannot end up at $I = 1$. (The remainder of the division $100/3$ is 1.)
2. No. The number of black squares minus the number of white squares is invariant and is initially equal to -2 . The final result, a fully covered board, requires that the invariant is equal to zero. Of course, this cannot happen. Make sure their proof is complete!
3. Every door has two “sides,” one toward one room and one toward either another room or the outside. Clearly the total number of sides, being twice the number of doors, is even. However, for each room, the number of sides pointing to it is even. Since even subtracted from even gives even, the number of sides pointing to the outside is also even.
4. The room problem.
 - (a) Yes. This is easy to show using simple guess and check.
 - (b) No. They don’t need to explain it using invariants yet, but let them struggle with it a bit or provide you a decent argument before you accept their answer that it is not possible.
 - (c) No. Consider what can happen in every increment of three minutes: the total number of people either increases by 3, stays constant, decreases by 3, or decreases by 6. This means that the remainder when divided by 3 stays constant. The remainder when divided by 3 is initially zero. After 999 minutes, the remainder must also be zero. So the “best” we can have is either 198 or 201 people in the room. It is impossible to get to 200 in the next minute from either of these positions.
 - (d) No. Blindly using the invariant would lead you to think that is possible. But, of course, we can only add one person per minute, and $3600 > 3000$.
5. Angela on a chessboard.
 - (a) yes. can be easily accomplished by drawing out a chessboard and making the necessary moves.
 - (b) No. (Never.) Consider the standard chess coloring of the board in black and white. An Angela always moves from a square of one color to a square of the same color. Because we know that two adjacent squares are always colored differently, the Angela cannot make the move.
6. Chessboards and polyminoes.

- (a) No. Consider the coloring given in the hint. For now, ignore the odd single piece. What do you notice about the parity of the number of white (or black) squares occupied by any of the 1×4 pieces? Now throw in the odd single piece. What happens?
 - (b) No. Consider the parity of white (or black) squares occupied by one polymino.
 - (c) No. Consider the difference between the number of occupied squares of any two colors: it must be divisible by 4. This is because we only have two options: we can place a polymino entirely in one row, where it adds 4 to that row's color, or entirely in one column, where it adds 1 to every row's color and leaves their difference unchanged. What is the initial difference in numbers of squares of each color in the 102×102 board? It's not divisible by 4!
7. 8. Since the remainders of a natural number and of the sum of its digits when divided by 9 are equal, the remainder of 8^{1989} coincides with the remainder of the final result x . Hence, x has remainder 8 when divided by 9, and we know that x is a digit, so $x = 8$.
 8. No. The invariant is the remainder when the number of heads is divided by 7.
 9. B. Consider the parities of the differences between the numbers of any two types. These are invariant. This means, in the end, the numbers of A and C have the same parity, which is possible if and only if the final amoeba is type B.
 10. No. Consider a quantity S , the sum of the number of heaps and stones. Have them show that S is invariant with an initial value of 1002. If there are only heaps of 3 stones remaining, then the total must be divisible by 4, which 1002 is not.