Propositional Logic

Math Circle (Advanced) 10/27/2013

The language of propositional logic consists of:
propositional variables: $P, Q, R, \ldots$
connective symbols $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
and punctuation: $(,)$**.**

0) Write down the meaning of the connective symbols:

- $\neg$:
- $\wedge$:
- $\vee$:
- $\rightarrow$:
- $\leftrightarrow$:

Well-formed formulas, also called wffs, are defined so that:

- Every propositional variable is a wff.
- If θ and λ are wffs, then so are

$$(\neg\theta),\ (\theta \wedge \lambda),\ (\theta \vee \lambda),\ (\theta \rightarrow \lambda),\ (\theta \leftrightarrow \lambda)$$

For example,

$$P,\ (P \rightarrow Q),\ ((P \vee Q) \rightarrow R),\ ((\neg P) \wedge (Q \leftrightarrow (\neg R)))$$

are all all wffs, while NONE of

$$\rightarrow P, PQ \rightarrow R, (P \vee \wedge R, P \vee Q \rightarrow R$$

are wffs.

1) Using the following notation (P ="it is raining", Q ="Jeff has an umbrella", and R = "Jeff is wet", translate the following statements into wffs (in propositional logic).

- It is raining.

- Jeff has an umbrella.

- Jeff is wet.

- If it is raining, then Jeff has an umbrella.

- Jeff is wet and it is raining.

- If it is raining and Jeff does not have an umbrella, then he is wet.

- It is raining, but Jeff does not have an umbrella, so he is wet.

- Jeff is wet but it is not raining if and only if Jeff has an umbrella or it is not raining.

2) Translate the following wffs into statements in English using the same statements from Problem 1.

- P

- Q

- R

- $(P \wedge Q) \rightarrow \neg R$

- $P \leftrightarrow (Q \vee R)$

- $P \rightarrow (Q \rightarrow P)$

- $(\neg Q \vee P) \rightarrow R$

3) Complete the following table:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
T	T					
T	F					
F	T					
F	F					

4) Suppose we know that P is T, Q is F, and R is T. (Note that this is called a *truth assignment* for P, Q, R.) What is the truth value of the following?

- $(P \wedge Q) \rightarrow R$
- $(\neg P \vee Q) \wedge (R \rightarrow Q)$
- $(Q \rightarrow (P \rightarrow R)) \leftrightarrow ((\neg Q \wedge P) \rightarrow R)$
- $[(\neg P \vee Q) \wedge (R \rightarrow Q)] \vee [(Q \rightarrow (P \rightarrow R)) \leftrightarrow ((Q \wedge P) \rightarrow R)]$

5) Translate the following into wffs, and then (using a truth table) decide if they are always true, always false, or neither.

- If there is smoke, then there is smoke.

- If there is smoke, then there is fire.

- There is smoke or there is fire, and there is not fire.

- If, if there is smoke there is fire, then there is smoke.

- Jeff is big and dumb, and if Jeff is big then he is not dumb.

- Jeff is big and dumb, if and only if Jeff is not big or not dumb.

6) An advertisement for a tennis magazine says: "If I'm not playing tennis, I'm watching tennis. And if I'm not watching tennis, then I'm reading about tennis." We can assume that the speaker can only do one of the above activities at a time. By translating the above into propositional logic, figure out what the speaker is doing.

7) Consider the following premise: You will get an extra credit point if you write a paper or if you solve the bonus test problem.

Your situation is as follows: Jeff didn't write a paper, but he got an extra credit point.

John asked, "Wow, how'd you solve that bonus test problem?!"

Jeff replied, "I didn't solve it."

Assign letters to each statement and form wffs to represent the situation. Is Jeff lying?

8) Consider the same premise as the previous problem, but now your situation is as follows:

Don didn't write a paper, and he didn't get an extra credit point.

Is it logically valid to conclude that Don did not solve the bonus test problem?