LAMC Beginners' Circle **December 8, 2013**

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Math Wrangle

3-point problems

Problem 1 *There are five apples in a basket. Split them between five children so that each child gets an apple and one apple is left in the basket.*

Problem 2 $\mathcal{A}$ *says, "I am a boy".* $\mathcal{B}$ *says, "I am a girl". If at least one of them is lying, who is a boy and who is a girl?*

Problem 3 *Oleg has drawn a few circles on a sheet of paper, put the sheet in front of Stella and asked to count the circles. Stella counted five circles. Then Oleg put the same sheet in front of Isaac and asked him to count the circles. Isaac counted seven circles. Both made no mistake. Is this possible?*

Problem 4 *Emily is \$7 short to buy 15 chocolates. If she buys 11 chocolates, she is left with \$5. Assuming that the chocolates cost the same, what is the price of one chocolate?*

Problem 5 *A ship is moored in a bay. The vessel has a rope ladder hanging down its side. The ladder has eleven steps, each step ten inches apart from the next one. The last step touches the water. In comes the tide raising the water level by 20 inches every hour. When would the water reach the third step from the top?*

Problem 6 *A son is twice as young as his father. The father was 24 when the son was born. How old is the son?*

4-point problems

Problem 7 *Put the signs $+$, $-$, $\times$, $\div$ and/or parenthesis between the digits below to produce a correct formula.*

$$4 \quad 4 \quad 4 \quad 4 = 5$$

Problem 8 *The sum of two natural numbers equals 474. The last digit of one of the numbers is one. If you mark out this digit, you will get the second number. What are the numbers?*

Problem 9 *Compute the following.*

$$1 - 3 + 5 - 7 + 9 - 11 + \ldots + 97 - 99 =$$

Problem 10 *Prove that given three integers, one can always choose two with an even sum.*

Problem 11 *1729 is the smallest positive integer that can be represented as a sum of two cubed positive integers in two different ways. Find the representations.*

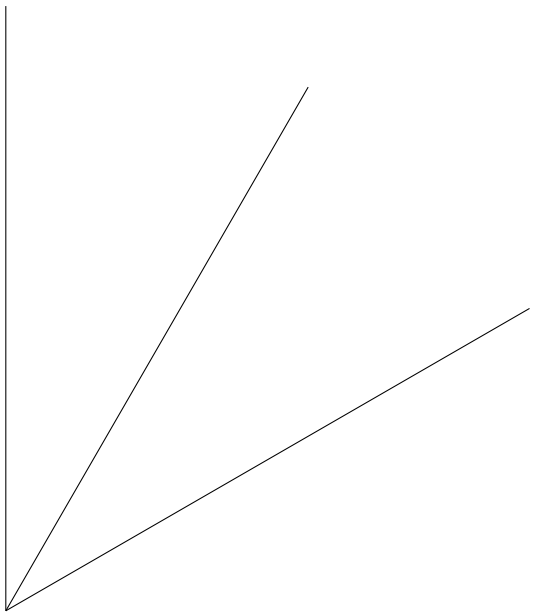
Problem 12 *Find $\sqrt{144_8}$ without switching to the decimals.*

5-point problems

Problem 13 *A son of the professor's father is talking to the father of the professor's son, but the professor does not take part in the conversation. Is this possible?*

Problem 14 *Can one put 24 chairs in six rows each having five chairs?*

Problem 15 *An angle bisectrix is a ray that originates at the angle's vertex and splits the angle into two congruent parts.*

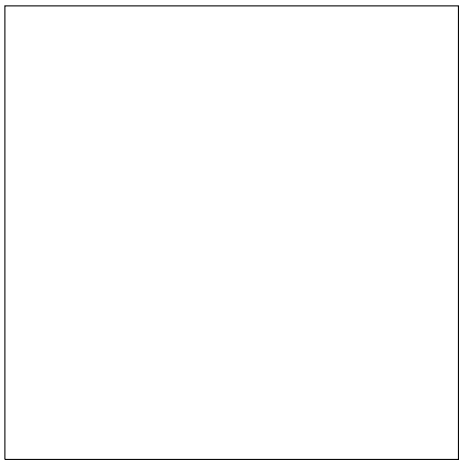


Prove that each point of the bisectrix is equidistant from the sides of the angle.

Problem 16 *Represent the number 186 as a sum of three different natural numbers such that each of the numbers divides the sum of the other two.*

Problem 17 *Anton took a number and subtracted the sum of its digits from the number. From the resulting number Anton subtracted the sum of its digits, and so on. The eleventh step of this computation yielded zero for the first time. What number did Anton begin with?*

Problem 18 *Split a square into two congruent pentagons.*



Captain's fight, 6-point problems

Problem 19 *All the* $40,000$ *squares on the* 200×200 *board are painted white. What is the maximal number of squares one can repaint black so that the black squares have no common points?*

Problem 20 *Prove that there exist a number* $11\ldots11$ *that is divisible by* 2013.