TRANSFORMATIONS VIA PERMUTATIONS

JUNIOR CIRCLE 05/08/2011

We will look at the following transformations of an equilateral triangle:

- Rotations;
- Reflections (flips) in a line;

The two types of rotation are:

- Clockwise rotation $\circlearrowright$:

- Counterclockwise rotation $\circlearrowleft$:

(1) When the triangle is rotated, the vertices end up in the new places. This way, we get a permutation of vertices:
- The first row is starting positions;
- The second row is ending positions;

Write down the permutations corresponding to the clockwise and the counterclockwise rotations:

(a) Clockwise rotation $\circlearrowright$:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix};$$

(b) Counterclockwise rotation $\circlearrowleft$:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix};$$

There are also three flips:

- The flip F_1 through line going through 1 and switching 2 and 3:

- The flip F_2 through line going through 2 and switching 1 and 3:

- The flip F_3 through line going through 3 and switching 1 and 2:

(2) When the triangle is flipped, the vertices also end up in the new places. Write down the permutations corresponding to the clockwise and the counterclockwise rotations:

(a) Flip F_1:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix};$$

(b) Flip F_2:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix};$$

(c) Flip F_3:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix};$$

When no transformation is performed, we get the *identity permutation:*

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 \end{pmatrix};$$

(3) Let's find the result of performing two transformations in a row:

(a) Find what transformation $\circlearrowleft \circ \circlearrowright$ equals to in two different ways:

- Label vertices and write down what the resulting transformation is:

- Multiply permutatations:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix};$$

- Are the results you get when using the picture and when multiplying permutations the same?

(b) Find what transformation $F_1 \circ F_3$ equals to in two different ways:

- Label vertices and write down what the resulting transformation is:

- Multiply permutations:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix};$$

- Are the results you get when using the picture and when multiplying permutations the same?

(c) Find what transformation $F_1 \circ \circlearrowleft$ equals to in two different ways:

- Label vertices and write down what the resulting transformation is:

- Multiply permutations:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ & & \end{pmatrix};$$

- Are the results you get when using the picture and when multiplying permutations the same?

(4) Fill out the following multiplication table (you can either draw pictures; or flip and rotate a model triangle, or multiply permutations).

	I	F_1	F_2	F_3	$\circlearrowright$	$\circlearrowleft$
I						
F_1						
F_2						
F_3						
$\circlearrowright$						
$\circlearrowleft$						

(5) Write down as many interesting things about this multiplication table as you can.

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