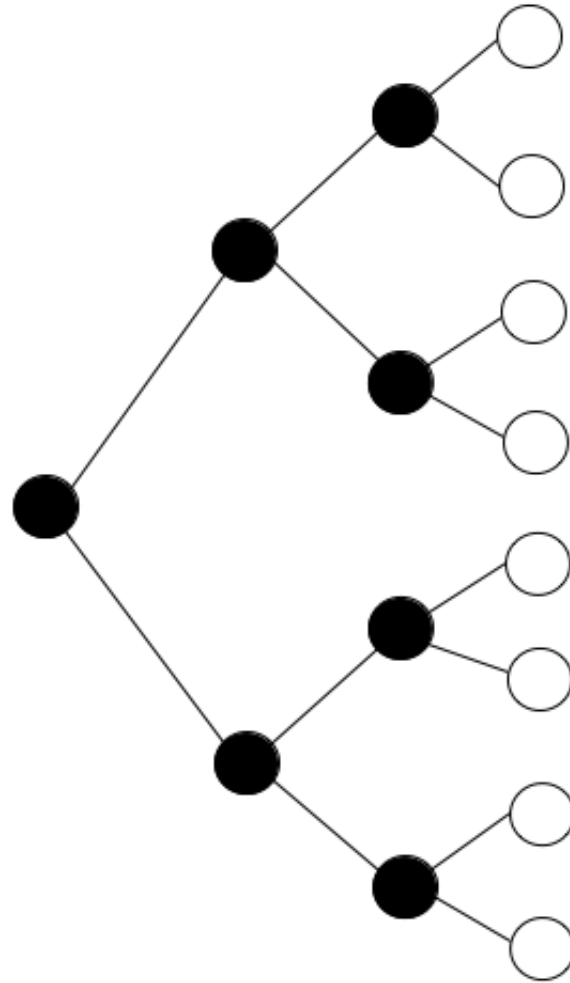


Binary Tree:



Binary Continued!

November 27, 2013

1. Label the vertices of the bottom row of your Binary Tree with the numbers 0 through 7 (going from left to right). (You may put numbers inside of the circles).

2. Play the game “Guess my number” (with numbers from 0 to 7). The goal is to be able to guess the number in 3 attempts (or less, if you are lucky). Can you think of how the edges can help you to formulate the strategy? What is the best way to play the game?

Solution: Always ask the question that will reduce the number of possible solutions by half until you are left with only one possible answer. So, start with the question, “Is it greater than 3?”, and depending on the answer, then ask, “Is it greater than five?”, or “Is it greater than 1?”, and so on and so forth. This will guarantee you to guess the number in three turns.

Label all the edges pointing to the left by 0; Label all the edges pointing to the right by 1. The path connecting the top circle with any of the numbers on the bottom gives you a string of 0s and 1s.

3. For each number on the bottom, there is exactly one path from the top circle down to this number.

a. Choose a color and mark the path from the top circle to the number 3.

b. Going from top to bottom, write down the string of 0s and 1s along the path going from the top circle to the number 3.

Solution: 011

4. Mark with a different color the path from the top circle to the numbers 6 and 7.

a. Write down the string of 0s and 1s along the path from the top circle to the number 6.

Solution: 110

b. Write down the string of 0s and 1s along the path from the top circle to the number 7.

Solution: 111

5. Do you recognize your answers in (3b), (4a), and (4b) (you can ignore the 0s in the beginning of the string)? What do these answers represent?

Solution: They are the binary representations of the corresponding numbers.

6. How many questions do you have to ask to guess the number in the “Guess my number” game with numbers ranging

a. from 0 through 7?

Solution: 3

b. from 0 through 15?

Solution: 4

c. from 0 through 31?

Solution: 5

A card trick

I have 4 cards with numbers 1 through 15 written on them. (Note that most of the numbers appear on several cards). Here are the cards:

Card 1	Card 2	Card 3	Card 4
$\begin{pmatrix} 8 & 9 & 10 & 11 \\ 12 & 13 & 14 & 15 \end{pmatrix}$	$\begin{pmatrix} 4 & 5 & 6 & 7 \\ 12 & 13 & 14 & 15 \end{pmatrix}$	$\begin{pmatrix} 2 & 3 & 6 & 7 \\ 10 & 11 & 14 & 15 \end{pmatrix}$	$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 9 & 11 & 13 & 15 \end{pmatrix}$

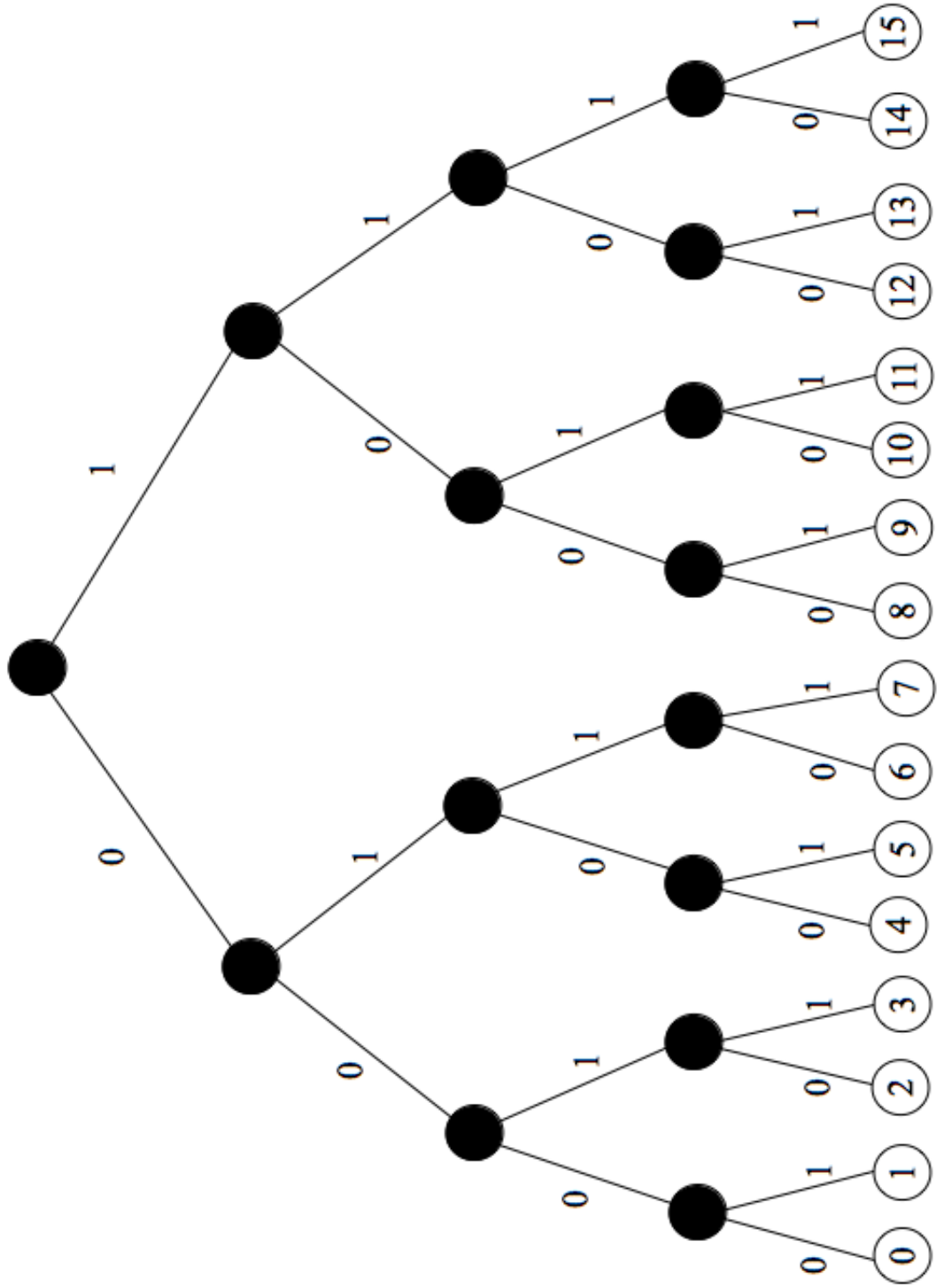
All these numbers can be written in binary notation using 4 digits (for some numbers, the first digit(s) can be 0s).

7. List all the numbers that appear only on one card. Do you recognize them?

Solution: 1, 2, 4, 8 (powers of 2)

8. What do all the numbers on Card 4 have in common?

Solution: They are all odd.



9. What do you notice about all the numbers on Card 1?

Solution: They are the eight largest numbers

10. Choose a color. On the binary tree given to you, circle the numbers appearing on Card 1.

a. Starting at the top circle, follow the paths you must take to reach the numbers on Card 1. What do all these paths have in common?

Solution: As you make your way down the tree, your first choice must be to follow an edge that has a 1 to get to any of these numbers..

b. What is the first binary digit of all the numbers appearing on Card 1?

Solution: 1

11. Choose a new color, and circle all the numbers appearing on Card 2.

a. Starting at the top circle, follow the paths you must take to reach the numbers on Card 2. What do all these paths have in common?

Solution: As you make your way down the tree, your second choice must be to follow an edge that has a 1 to get to any of these numbers..

b. What is the second binary digit of all the numbers appearing on Card 1?

Solution: 1

12. Choose a third color, and circle all the numbers appearing on Card 3.

a. Starting at the top circle, follow the paths you must take to reach the numbers on Card 3. What do all these paths have in common?

Solution: As you make your way down the tree, your third choice must be to follow an edge that has a 1 to get to any of these numbers..

b. What is the third binary digit of all the numbers appearing on Card 3?

Solution: 1

13. Choose a fourth color, and circle all the numbers appearing on Card 4.

a. Starting at the top circle, follow the paths you must take to reach the numbers on Card 4. What do all these paths have in common?

Solution: As you make your way down the tree, your fourth choice must be to follow an edge that has a 1 to get to any of these numbers.

b. What is the fourth binary digit of all the numbers appearing on Card 4?

Solution: 1

14. Is there a number that appears on all the cards? How do you write that number in binary notation?

Solution: 15 appears on all the cards. In binary notation, that is 1111.

Homework!

Review all the material we learned this Fall and get ready for the last day of class! :)