

Adding and Subtracting Binary

Explain how you solve the following addition problem in decimal notation:

$$\begin{array}{r} + \quad 6 \ 4 \\ \quad 4 \ 5 \\ \hline 1 \ 0 \ 9 \end{array}$$

Here are the addition facts that you need when adding numbers in binary notation. Explain these addition facts by converting numbers to decimal notation. The first one is done for you:

Binary Notation

Decimal Noation

* $\boxed{0} + \boxed{0} = \boxed{0}$

$\underline{0} + \underline{0} = \underline{0}$

* $\boxed{0} + \boxed{1} = \boxed{1}$

Solution: $0+1=1$

* $\boxed{1} + \boxed{1} = \boxed{1} \boxed{0}$

Solution: $1+1=2$

1. Explain which of these addition facts will lead to carrying when you are adding bigger numbers. Now explain this problem in binary notation:

Solution: The fact that $\boxed{1} + \boxed{1} = \boxed{1} \boxed{0}$ will lead to carrying since the only digits in binary are $\boxed{0}$ and $\boxed{1}$.

$$\begin{array}{r} + \quad \boxed{1} \ \boxed{0} \\ \quad \boxed{1} \ \boxed{1} \\ \hline \boxed{1} \ \boxed{0} \ \boxed{1} \end{array}$$

2. Solve the following addition problems (be careful as carrying may be involved). Remember to box your solutions! Check your work by solving the problem in decimal notation on the side.

Binary Notation

Decimal Notation

a.)
$$\begin{array}{r} \boxed{1} \boxed{0} \boxed{1} \boxed{0} \\ + \boxed{1} \boxed{0} \boxed{1} \\ \hline \end{array}$$

Solution: $\boxed{1} \boxed{1} \boxed{1} \boxed{1}$

b.)
$$\begin{array}{r} \boxed{1} \boxed{1} \boxed{1} \boxed{1} \\ + \boxed{1} \boxed{1} \boxed{1} \boxed{0} \\ \hline \end{array}$$

Solution: $\boxed{1} \boxed{1} \boxed{1} \boxed{0} \boxed{1}$

$$\begin{array}{r} \boxed{1} \boxed{0} \boxed{1} \boxed{0} = _ \\ \boxed{1} \boxed{0} \boxed{1} = _ \\ _ + _ = _ \\ (10 + 5 = 15) \end{array}$$

$$\begin{array}{r} \square \square \square \square = _ \\ \square \square \square \square = _ \\ _ + _ = _ \\ (15 + 14 = 29) \end{array}$$

Now, let's try subtracting. Here again are some standard subtraction facts we know.

Binary Notation

Decimal Noation

$$* \boxed{1} - \boxed{1} = \boxed{0}$$

Solution: $1-1=0$

$$* \boxed{1} - \boxed{0} = \boxed{1}$$

Solution: $1-0=1$

$$* \boxed{1} \boxed{0} - \boxed{1} = \boxed{1}$$

Solution: $2-1=1$

3. Explain which of these subtraction facts will lead to borrowing when you are subtracting bigger numbers. (Hint: think about why you have to borrow in decimal notation!) Now explain this problem in binary notation:

Solution: The fact that $\boxed{1}\boxed{0} - \boxed{1} = \boxed{1}$ will lead to borrowing since you can not take $\boxed{1}$ from $\boxed{0}$ for a certain place value, and so you must borrow from the $\boxed{1}$ in the next place value, much like how you need to borrow in decimal notation.

$$\begin{array}{r}
 \boxed{1} \ \boxed{0} \ \boxed{0} \\
 - \quad \boxed{1} \ \boxed{0} \\
 \hline
 \quad \boxed{1} \ \boxed{0}
 \end{array}$$

4. Subtract the following binary numbers:

Binary Notation

a.)

$$\begin{array}{r} \boxed{1} \boxed{1} \boxed{1} \boxed{1} \\ - \boxed{1} \boxed{0} \boxed{0} \boxed{1} \\ \hline \end{array}$$

Solution: $\boxed{1} \boxed{1} \boxed{0}$

Decimal Notation

$$\begin{array}{r} \square \square \square \square = _ \\ \square \square \square \square = _ \\ \hline _ - _ = _ \end{array}$$

(15 - 9 = 6)

b.)

$$\begin{array}{r} \boxed{1} \boxed{0} \boxed{0} \boxed{1} \\ - \quad \boxed{1} \boxed{1} \boxed{0} \\ \hline \end{array}$$

Solution: $\boxed{1} \boxed{1}$

$$\begin{array}{r} \square \square \square \square = _ \\ \square \square \square \square = _ \\ \hline _ - _ = _ \end{array}$$

(9 - 6 = 3)

Counting Binary Numbers with your Fingers!

5. Last week, we learned to count to 31 with your fingers of your left hand. For the pictures of different hand positions below, write down the numbers they represent in binary notation and decimal notation.

Remember that the pinky finger is the highest digit (16) and the thumb is the lowest digit (1).



a.

Solution: 3



b.

Solution: 14



c.

Solution: 2



d.

Solution: 17

Back to binary adding! Solve the following addition problems. Remember to box your solutions.

$$\begin{array}{r}
 \boxed{1} \boxed{0} \boxed{0} \boxed{0} \boxed{1} \\
 \boxed{1} \boxed{0} \boxed{0} \boxed{1} \\
 + \boxed{1} \boxed{0} \boxed{1} \\
 \hline
 \end{array}$$

6.

Solution: $\boxed{1} \boxed{1} \boxed{1} \boxed{1} \boxed{1}$ ($17+9+5=31$)

$$\begin{array}{r}
 \boxed{1} \boxed{0} \boxed{1} \boxed{0} \boxed{1} \\
 \boxed{1} \boxed{0} \boxed{1} \boxed{0} \\
 + \boxed{1} \\
 \hline
 \end{array}$$

7.

Solution: $\boxed{1} \boxed{0} \boxed{0} \boxed{0} \boxed{0} \boxed{0} \boxed{0}$ ($21+10+1=32$)

8. Compute the sum $1+1+2+4+8+16+32$ like this: first, convert all numbers to binary. Then carry out the addition. Last, convert to a decimal number. (It is not a typo, there are two 1s in this sum!).

Solution:

$$\begin{array}{r}
 \square \quad \square \quad \square \quad \square \quad \square \quad \square \quad 1 \\
 \square \quad \square \quad \square \quad \square \quad \square \quad \square \quad 1 \\
 \square \quad \square \quad \square \quad \square \quad \square \quad 1 \quad 0 \\
 \square \quad \square \quad \square \quad \square \quad 1 \quad 0 \quad 0 \\
 \square \quad \square \quad \square \quad 1 \quad 0 \quad 0 \quad 0 \\
 \square \quad \square \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \\
 \square \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
 + \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad =64
 \end{array}$$