More Adventures in Binary Land

4/15/12

Decimal Notation:

1. What is the value of the digit 8 in the number 938?

Solution: 8, (8 ones)

2. What is the value of the digit 8 in the number 581?

Solution: 80, (8 tens)

3. What is the value of the digit 8 in the number 824?

Solution: 800 (8 one hundreds)

Let's look at the number 3456:

4. What is the value of the digit 3?
Solution: 3000 (3 One Thousands)
5. What is the value of the digit 5?
Solution: 50 (5 Tens)
6. What is the value of the digit 4?
Solution: 400 (4 Hundreds)

Binary Notation:

7. Put the numbers 7, 15, 20, 29, 36, 43, and 50 into binary notation by filling out the table below:

(a) What is the value of the rightmost digit 1 in the binary notation of the number 7?

Solution: 1

(b) What is the value of the rightmost digit 1 in the binary notation of the number 20?

Solution: 4

(c) What is the value of the rightmost digit 1 in the binary notation of the number 36?

Solution: 4

Binary Counting with your fingers!
Each finger represents a binary digit of a number. Have your instructor label your fingers for you.

If a finger is up, the digit is 1; if a finger is down, the digit is 0. (For the best results, put your hand flat against the table to keep the finger position.)

Write the following numbers in binary notation, with the help of your fingers.

8. $5 = 101$
9. $9 = 1001$
10. $17 = 10001$
11. $28 = 11100$

12. What is the highest number you can count to on the fingers of one hand in binary notation?
Solution: 31

13. For each of the binary numbers, implement them on your hand (press the hand on the table as you are doing this to make it easier). Then add together the values of all the digits to get the decimal number:

a.) $\boxed{1}\boxed{1} = 3$

b.) $\boxed{1}\boxed{0}\boxed{0}\boxed{0}\boxed{1} = 17$

c.) $\boxed{1}\boxed{0}\boxed{1}\boxed{1}\boxed{1} = 23$

d.) $\boxed{1}\boxed{1}\boxed{0}\boxed{1}\boxed{1} = 27$

e.) $\boxed{1}\boxed{0}\boxed{1}\boxed{0}\boxed{1} = 21$

Explain the following addition problem in decimal notation:

$$\begin{array}{rrr} & \overset{\cdot}{5} & 4 \\ + & 4 & 6 \\ \hline 1 & 0 & 0 \end{array}$$

Now look at a similar problem in binary notation.

$$\begin{array}{rrrr} & & \overset{\cdot}{1} & 1 \\ + & & & 1 \\ \hline & 1 & 0 & 0 \end{array}$$

Here are the addition facts that you need when additing numbers in binary notation.Explain these addition facts by converting numbers to decimal notation:

* $0 + 0 = 0$
* $0 + 1 = 1$
* $1 + 1 = 10$

14. Explain which of these addition facts will lead to carrying when you are adding bigger numbers. (Hint: think about why you have to carry in decimal notation!)

15. Solve the following addition problems (Be careful as carrying may be involved). Check your work by solving the problem in decimal notation on the side.

a.)
$$\begin{array}{r} 1\,0\,1\,0 \\ +\quad 1\,0\,1 \\ \hline \end{array}$$

Solution: 1111 (15)

b.)
$$\begin{array}{r} 1\,0\,1\,1 \\ +\;1\,0\,1\,0 \\ \hline \end{array}$$

Solution: 10101 (21)

c.)
$$\begin{array}{r} 1\,1\,1\,1 \\ +\;1\,1\,1\,0 \\ \hline \end{array}$$

Solution: 11101 (29)

Now, let's try subtracting. Here again are some standard subtraction facts we know.

* $1 - 1 = 0$
* $1 - 0 = 1$
* $10 - 1 = 1$

16.Subtract the following binary numbers:(Solutions in decimal notation are in parentheses)

a.)
$$\begin{array}{r} 1111 \\ -\ 1001 \\ \hline \end{array}$$

Solution: 110 (6)

b.)
$$\begin{array}{r} 1001 \\ -\ \ 110 \\ \hline \end{array}$$

Solution: 11 (3)

c.)
$$\begin{array}{r} 1011 \\ -\ \ \ 11 \\ \hline \end{array}$$

Solution:1000 (8)

d.)
$$\begin{array}{r} 1101 \\ -\ \ \ 11 \\ \hline \end{array}$$

Solution: 1010 (10)

17. Compute the sum 1+1+2+4+8+16+32 like this: first, convert all numbers to binary. Then carry out the addition. Last, convert to a decimal number. (It is not a typo, there are two 1s in this sum!).

Solution:

$$
\begin{array}{cccccccc}
 & \square & \square & \square & \square & \square & \square & 1 \\
 & \square & \square & \square & \square & \square & \square & 1 \\
 & \square & \square & \square & \square & \square & 1 & 0 \\
 & \square & \square & \square & \square & 1 & 0 & 0 \\
 & \square & \square & \square & 1 & 0 & 0 & 0 \\
 & \square & \square & 1 & 0 & 0 & 0 & 0 \\
 & \square & 1 & 0 & 0 & 0 & 0 & 0 \\
+ & 1 & 0 & 0 & 0 & 0 & 0 & 0 \quad =64
\end{array}
$$

HOMEWORK:

1.Create your own problem about binary notation. Write your problem here (in the handout) and solve it. Then, write the problem on an index card. Next time, you will exchange with a partner and solve each other's problems.

2. Katja is 23 years old, Travis is 22 years old, and April is 19 years old. Write their ages in binary:

Solution:

Katja: 10111

Travis:10110

April:10011