

# An application of Combinatorics

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## Questions we worked through

These questions should be answered using choose notation!

1. In the middle of the night Derek began sleepwalking. He left his room, into his (infinitely long!) hallway. His sleepwalking is very random but he always moves in a straight line. Every minute he either steps forward or backward.
  - (a) How many ways are there for Derek to return home after 17 minutes?  $0$
  - (b) How many ways are there for Derek to return home after 6 minutes?  $\binom{6}{3}$
  - (c) How many ways are there for Derek to return home after 10 minutes?  $\binom{10}{5}$
2. Dustin lives 4 steps away from Derek and hears him walking around outside. He see's Derek directly outside his own door and puts a blocker outside his door. (So now Derek cannot walk past Dustin's door!) [Here we will assume that he has to go forward after going backward 4 times. You can do the case where getting a backwards does nothing yourself!]
  - (a) How many ways are there for Derek to return home after 17 minutes?  $0$
  - (b) How many ways are there for Derek to return home after 6 minutes?  $\binom{6}{3}$
  - (c) How many ways are there for Derek to return home after 10 minutes?  $\binom{10}{5} - 1$
3. Derek has safely returned to his room, and gotten ready for the day. Morgan, however, had a crazy night and has woken up lost in an empty field. Every second he steps either Right, Forward, Backward, or Left.
  - (a) How many ways are there for Morgan to return where he started after 15 minutes?  $0$
  - (b) How many ways are there for Morgan to return after 16 minutes?  $\binom{16}{8}^2$  (This is hard, we proved it with mountains)

We will work to answer this ridiculously difficult question via an identification.

## How many ways for Morgan to return after 16 minutes?

4. To solve this we'll look at the number of ways for Pax to do something a bit different. Pax is drawing mountain ranges of size 16! He draws mountain ranges by drawing a diagonal lines. He either draws

a line up and to the right (north east) or down and to the right (southeast). A mountain range starts and ends at the same height. You should remember what these look like. Here is space for you to draw one yourself:

(a) How many ways are there for Pax to draw a mountain range of length 16?  $\binom{16}{8}$

(b) How many ways are there for Pax to draw two mountain ranges of length 16 each?  $\binom{16}{8}^2$ .

5. Now that we know how to help Pax count, lets try to find a way to identify this with Morgan's situation.

(a) We look at Pax's two mountain ranges simultaneously. In each position  $1, 2, 3, \dots, 16$  we get a double (Up, Up), (Up, Down), (Down, Up), (Down, Down).

(b) We can make an identification as follows:

$$Left = (Up, Up), Right = (Down, Down), Up = (Up, Down), Down = (Down, Up)$$

i. Draw some Pax mountains and find the Morgan equivalent:

ii. Draw some Morgan Walks and find the Pax equivalent:

(c) How many ways are there for Morgan to return after 16 minutes?  $\binom{16}{8}^2$

- We saw that there are the same number of Morgan walks as there Pax mountains. And there are  $\binom{16}{8}^2$  Pax mountains!