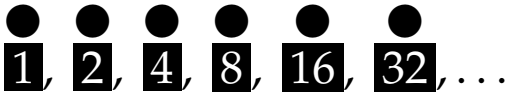
Weighing with Powers of 2

In the country of Binary Land, the factory makes only the following weights (in kilograms):



You can buy any number of these standard weights from the factory and use them with a balance scale.

1. What is the pattern in the sequence of weights above? **Solution: The weights double from one to the next.**

2. Write down the next 3 weights in the sequence: **Solution: 64, 128, 256**

3. Balance each of the following objects using the weights

   (a) Find two different ways to balance a watermelon weighting 7 kilograms. (Note that you can use the same weight more than once). **Solution: 4+2+1, 2+2+2+1, etc...**

   (b) Find two different ways to balancs a metal ball weighing 10 kilograms. (Note that you can use the same weight more than once). **Solution: 8+2, 2+2+2+2+2, etc**..

4. Now try to use each of the standard weight only once. Can you balance the following:

   (a) **Solution: 4+2**

   (b) **Solution: 8+4+1**

   (c) **Solution: 16+1**

   (d) **Solution: 16+1**

   (e) **Solution: 16+8**

5. Balance the following. Do not use the same weight more than once. Fill in the table below:

   - if you are using a certain weight, put 1 in its column;
   - if you are not using a certain weight, put 0 in its column.

   Some examples are filled in:

Weight	Write as sum of weights	8	4	2	1
1	$1 = 1$	0	0	0	1
2	$2 = 2$	0	0	1	0

Weight	Write as sum of weights	8	4	2	1
3	3 =2+1	0	0	1	1
4	4 =4	0	1	0	0
5	5 =4+1	0	1	0	1
6	6 =4 2	0	1	1	0
7	7 =4+2+1	0	1	1	1
8	8 =8	1	0	0	0
9	9 =8 1	1	0	0	1
10	10 =8+2	1	0	1	0
11	11 =8+2+1	1	0	1	1
12	12 =8+4	1	0	1	0
13	13 =8 4 1	1	1	0	1
14	14 =8+4+2	1	1	1	0
15	15 =8+4+2+1	1	1	1	1

6. Major robbery!

   (a) After many weights were stolen you are left only with the weights 1, 2, 4 and 8. Can you balance 18 kilograms using each weight no more than once? Why or why not? What weights can you balance? **Solution: No since the biggest number you can get is 15<18. You can balance the ones from the previous page.**

   (b) What is the largest number you can write as a sum of some the numbers $1, 2, 4, 8$ without using the same number more than once? **Solution:8+4+2+1=15**

(c) Can you balance 30 kilograms with the given weights

$$1, 2, 4, 8, 16, \ldots$$

without using any weight more than once. That is, write 30 as a sum of several of these numbers, without repeating any of the numbers:**Solution:**

$$\mathbf{30 = 16 + 8 + 4 + 2}.$$

Now write down the string of 0s and 1s that corresponds to 30. (Write 1 under the numbers that are used in the sum above. Write 0 under the numbers that are not used in the sum above):

	16	8	4	2	1
30	1	1	1	1	0

(d) Can you write down 57 as sum of some of the numbers

$$1, 2, 4, 8, 16, 32, \ldots?$$

Do not use the same number twice.

$$\mathbf{57 = 32 + 16 + 8 + 1}.$$

Now write down the string of 0s and 1s that corresponds to 57:

	32	16	8	4	2	1
57	1	1	1	0	0	1

7. What number corresponds to the following strings of 0s and 1s. Fill in the table:

32	16	8	4	2	1	computation	number
0	1	1	0	1	0	$16+8+2=26$	26
0	1	1	0	1	0	**16+8+2=26**	**26**
0	0	1	1	0	1	**8+4+1**	**13**
1	1	0	0	1	0	**32+16+2**	**50**
1	1	1	1	1	1	**32+16+8+4+2+1**	63

8. Every number can be written in *binary notation:*

$$1 = \boxed{1}, \qquad 2 = \boxed{1}\boxed{0}, \qquad 3 = \boxed{1}\boxed{1}, \qquad 4 = \boxed{1}\boxed{0}\boxed{0}.$$

Find the missing numbers or binary notation:

$\boxed{1}\boxed{0}\boxed{1} = \mathbf{5}$ $\qquad$ $\boxed{1}\boxed{1}\boxed{0} = \mathbf{6}$ $\qquad$ $\boxed{1}\boxed{0}\boxed{0}\boxed{1} = \mathbf{9}$

$\mathbf{110} = 6$ $\qquad$ $\mathbf{111} = 7$ $\qquad$ $\mathbf{100} = 8$

Here the numbers where each digit is written inside of a square are written in binary notation.