

# Weighing with Powers of 2

In the country of Binary Land, the factory makes only the following weights (in kilograms):

$\overset{\bullet}{\boxed{1}}, \overset{\bullet}{\boxed{2}}, \overset{\bullet}{\boxed{4}}, \overset{\bullet}{\boxed{8}}, \overset{\bullet}{\boxed{16}}, \overset{\bullet}{\boxed{32}}, \dots$

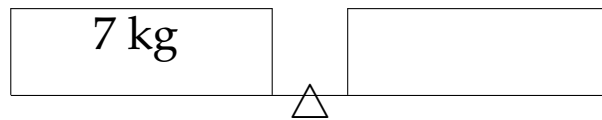
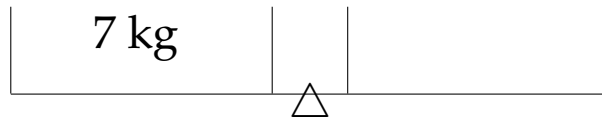
You can buy any number of these standard weights from the factory and use them with a balance scale.

1. What is the pattern in the sequence of weights above?
2. Write down the next 3 weights in the sequence:

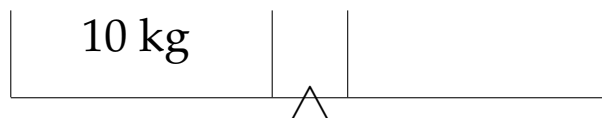
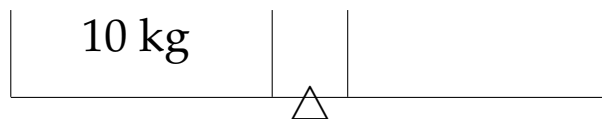
3. Balance each of the following objects using the weights

●  
**1**, ●  
**2**, ●  
**4**, ●  
**8**, ●  
**16**, . . . .

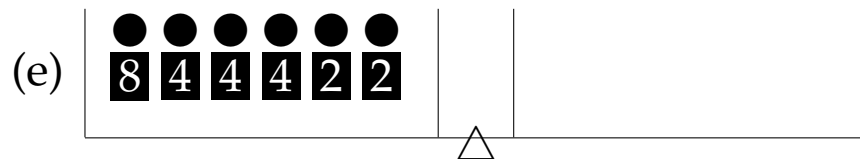
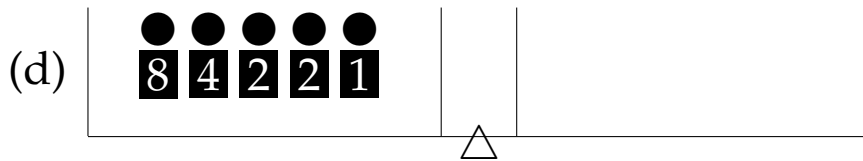
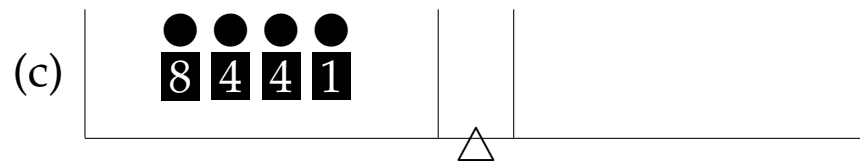
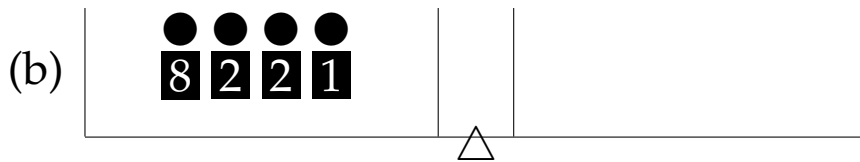
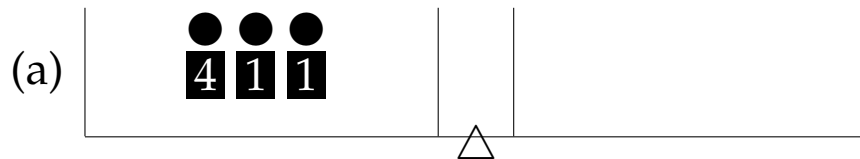
(a) Find two different ways to balance a watermelon weighting 7 kilograms. (Note that you can use the same weight more than once).



(b) Find two different ways to balance a metal ball weighing 10 kilograms. (Note that you can use the same weight more than once).
















4. Now try to use each of the standard weight only once.  
Can you balance the following:



5. Balance the following. Do not use the same weight more than once. Some examples are filled in. Can you fill in the rest?

- if you are using a weight, put 1 in its column.
- if you are not using a weight, put 0 in its column.

Weight	Write as sum of weights				
1	1 = 	0	0	0	1
2	2 = 	0	0	1	0
3	3 =				
4	4 =	0	1	0	0
5	5 =				
6	6 =  	0	1	1	0
7	7 =				
8	8 =				
9	9 =  	1	0	0	1
10	10 =				
11	11 =				
12	12 =				
13	13 =   	1	1	0	1
14	14 =				
15	15 =				

6. Major robbery!

(a) After many weights were stolen you are left only with the weights  $\overset{\bullet}{1}$ ,  $\overset{\bullet}{2}$ ,  $\overset{\bullet}{4}$  and  $\overset{\bullet}{8}$ . Can you balance 18 kilograms using each weight no more than once? Why or why not? What weights can you balance?

(b) What is the largest number you can write as a sum of some the numbers 1, 2, 4, 8 without using the same number more than once?

7. Can you balance 30 kilograms with the given weights

$$1, 2, 4, 8, 16, \dots$$

without using any weight more than once?

$$30 = \square + \square + \square + \square.$$

Now write down the string of 0s and 1s that corresponds to 30. (Write 1 under the numbers that are used in the sum above. Write 0 under the numbers that are not used in the sum above):

	16	8	4	2	1
30					

(c) Can you write down 57 as sum of some of the numbers

$$1, 2, 4, 8, 16, 32, \dots?$$

Do not use the same number twice.

$$57 = \square + \square + \square + \square.$$

Now write down the string of 0s and 1s that corresponds to 57:

	32	16	8	4	2	1
57						

7. What number corresponds to the following strings of 0s and 1s. Fill in the table:

32	16	8	4	2	1	computation	number
0	1	1	0	1	0	$16 + 8 + 2 = 26$	26
0	1	1	0	1	0		
0	0	1	1	0	1		
1	1	0	0	1	0		
1	1	1	1	1	1		

8. Every number can be written in *binary notation*:

$$1 = \boxed{1}, \quad 2 = \boxed{10}, \quad 3 = \boxed{11}, \quad 4 = \boxed{100}.$$

Find the missing numbers or binary notation:

$$\begin{array}{ccc} \boxed{101} = & \boxed{110} = & \boxed{1001} = \\ \square\square\square = 6 & \square\square\square = 7 & \square\square\square = 8 \end{array}$$

Here the numbers where each digit is written inside of a square are written in binary notation.