LAMC Beginners' Circle **October 20, 2013**

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Warm-up

Problem 1 *Use four fours to make thirty nine.*

Pigeonhole principle

If there are $n + 1$ pigeons in n holes, then there exist at least one hole with two pigeons.

Problem 2 *Is it possible to cover an equilateral triangle with two smaller equilateral triangles? Why or why not?*

Problem 3 *You are given 224 integers. Prove that there exist at least two of them such that their difference is divisible by 223.*

Problem 4 *You are given a* (8×8) *chess board with a pair of opposite corner squares cut off. You are further given a set of dominoes each equal in size to a pair of the board squares with a common side. Is it possible to tile the board with the dominoes in such a way that all the board squares are covered while the dominoes neither overlap nor stick out?*

Problem 5 *The ocean covers more than a half of the Earth's surface. Prove that the ocean has at least one pair of antipodal points.*

Problem 6 *There are* $n > 1$ *people at a party. Prove that among them there are at least two people who have the same number of acquaintances at the gathering. (We assume that if A knows B, then B also knows A.)*

Problem 7 *Among any five points with integer coordinates in the plane, there exist two such that the center of the line segment connecting them has integer coordinates as well.*

Problem 8 *Prove that if every point on a straight line is painted either black or white, then there exist three points of the same color such that one is the midpoint of the line segment formed by the other two.*

Problem 9 *All the points in the plain are painted with either one of two colors. Prove that there exist two points in the plain that have the same color and are located exactly one foot away from each other.*

Problem 10 *Let n be an integer not divisible by 2 and 5. Show that n has a multiple consisting entirely of ones.*

Problem 11 *Prove that for any $n > 1$, there exists an integer made of only sevens and zeros that is divisible by n.*

Problem 12 *Let n be an odd number. Let $a_1, a_2, \ldots, a_n$ be a permutation of the numbers $1, 2, \ldots, n$. Prove that the product $(a_1 - 1) \times (a_2 - 2) \times \ldots \times (a_n - n)$ is an even number.*

Problem 13 *A stressed-out UCLA student consumes at least one espresso every day of a particular year, drinking 500 overall. Prove that on some consecutive sequence of whole days the student drinks exactly 100 espressos.*

Problem 14 *Prove that at a party with ten or more people, there are either three mutual acquaintances or four mutual strangers.*

The following is often presented as a problem on the pigeonhole principle. I (O.G.) have found a solution not involving the principle, but failed to find a solution that utilizes one. If you find such a solution, please let me know.

Problem 15 *Given a table with a marked point, O, and with 2013 properly working watches put down on the table, prove that there exists a moment in time when the sum of the distances from O to the watches' centers is less than the sum of the distances from O to the tips of the watches' minute hands.*