FROM DECIMALS TO FRACTIONS

LAMC INTERMEDIATE GROUP - 10/13/13

WARM UP!

Deven was a bit curious about what the other instructors favorite numbers were. Unfortunately the Math Circle instructors are very tricky and gave Deven the information in a way that thoroughly confused him. Help Deven figure out each instructors favorite number! Of course, Deven would like to know their favorite numbers in both fraction and decimal form.

- Dustin, as the newest Math Circle instructor, provided Deven with the least tricky information, stating "If you multiply my number by 5, then add 1 you get 5." What is Dustin's favorite number?

- Pax says "If you multiply my favorite number by 10, you get the same result as you would if you added 6 to it." What is Pax's favorite number?

- If you multiply Derek's favorite number by 100, you get the same result as you would if you added 100 to Derek's number. What is Derek's number?

- Finally, since Morgan is the trickiest of the instructors he says "If you multiply my number by 1000, you get the same result as if you added 37 to it." What is Morgan's number?

(1) We already proved that if a fraction has a denominator whose only prime factors are 2 and 5, then the decimal equivalent terminated. Our goal to begin this week is to prove this is the only way a fraction can have a terminating decimal. Morgan, who never likes to do more work than he has to, claims "There isn't anything to prove at all, we've already done this." Is he right? Why or why not?

(2) Convert the following decimals into fractions in lowest terms. Pay special attention to the prime factorization of the denominators.

(a) .89

(b) .35

(c) .0125

(d) 44.44

(3) Show that given an arbitrary terminating decimal we can multiply it by 10^k for some k to get an integer. (*Hint: look at the previous problem for some examples!)*

(4) Show that given an arbitrary terminating decimal it has a fractional equivalent with denominator with only powers of 2 and 5. (*Hint: use the previous problem!)*

(5) Convert the following fractions into decimals–do you see a pattern? Complete the pattern with the next few fractions and their decimal representations.

(a) 1/9

(b) 2/9

(c) 3/9

(d) 4/9

(e) 5/9

(f) 6/9

(g) 7/9

(h) 8/9

(i) Give an example of a number with two distinct decimal representations. Prove the two decimals represent the same number.

(6) Building on the pattern you saw in the previous problem, see if you can convert these infinite decimals into fractions:

(a) $3.33333333\ldots$

(b) $17.55555555\ldots$

(c) $2.0088888888\ldots$

(7) Consider the decimal number $x = 0.22222222\cdots$, which has 1 repeating digit.

(a) What is $10x$ as a decimal?

(b) Write $10x$ as x plus some number (**NOT** $10x = x + 9x$)?

(c) Solve the equation you got in (b) for x.

(d) Based on part (c), what is the representation of x as a fraction?

(8) Try using the process from Problem 11 to write $x = 0.66666666\cdots$ as a fraction. (Write down the steps of the process even if you already know the answer!)

(9) Now let's try a decimal like $w = 0.466666666\cdots$, which starts with a 4 and then repeats the digit 6.

(a) What is $10w$ - 4 as a decimal?

(b) What is $10w$ - 4 as a fraction?

(c) Use part (b) to help you write w itself as a fraction.

(10) Consider the number $y = 0.36363636\cdots$, which has 2 repeating digits.

(a) What is $100y$ as a decimal?

(b) Write $100y$ as y plus some number.

(c) Solve the equation you got in (b) for y?

(d) Based on part (c), what is the representation of y as a fraction?

(11) Try using the process from Problem 12 to write $y = 0.05050505\cdots$ as a fraction.

(12) Consider the decimal $z = 0.148148148148\cdots$, which has 3 repeating digits. Use a similar process to that in Problems 11 and 14, to write z as a fraction.

(13) The decimal $0.abcabcabcabc\cdots$ has 3 unknown repeating digits. How could you write it as a fraction? (Your fraction will have to use a, b, and c to represent the unknown digits also! It does not have to be in lowest terms.)

(14) A number is called **rational** if it can be written in the form $\frac{a}{b}$ where a and b are integers.

(a) Is 5242362342 rational? Why or why not?

(b) What about -47.26? Why or why not?

(c) Is every eventually periodic (*recall the definition from last week!*) decimal rational?

(d) Is every decimal rational? Why or why not?

HOMEWORK!

(1) Are the following numbers rational?

(a)

$$\frac{.0101001000\cdots}{1}$$

(b)

$$\frac{\frac{13}{7^2}\cdot 2^2\cdot \frac{14}{9}}{55^3\cdot 19}$$

(2) Can you write down two numbers, a and b such that neither a nor b is rational but $a+b$ is? If not, why not? If yes, do it.

(3) Convert the following decimal into a fraction: $.defabcabcabc\cdots$, that is it begins with def then simply repeats abc.