Find the Fake Coin

Meeting 3

October 9, 2011

From last week...

1. Lisa puts 5 grapefruit on one side of the scale. There is a 2 pound weight on the other side of the scale.

a. Is the scale balanced? Why or why not?

Solution: No, since the pumpkins add up to 5 lbs, and 5>2

b. Can Lisa add a watermelon or a pumpkin to one of the sides to balance the scale ?

Solution: Yes, Lisa can add a pumpkin to the side of the 2 lb weight.

2.Suppose you have 3 coins. One of them is fake and is lighter than the other two. Use the balance scale to find the fake coin. How many tries do you need to use?

Solution: 1 try

b.)Can you find the fake coin with just one try? If so, how?

Solution: Yes, try 1 coin against another one. If they are the same, you know the third coin is the fake one. If they are not the same, you know the lighter one is the fake one.

3.

a. Now you have 9 coins, and one of them is fake. The fake coin is lighter than the rest. Can you come up with a method that will always find the fake coin? How?

Solution: Yes

b. Can you find the fake coin with only 2 tries?

Solution: Yes, break the 9 coins into three groups of three. Try two groups of three against each other. If they are the same, you know the fake coin is in the third pile. If one is lighter, you know the fake coin is in that pile. Now take the pile which has the fake coin, and repeat what you did in the previous problem.

4. What if you have 12 coins and one of them is fake? You know that the fake coin is lighter than the real coins. What is the smallest amount of tries you need to use to make sure you know which one is fake?

Solution: 3 tries. There are several different ways to do this. Some options: break them into groups of 2 groups of 6, then 2 groups of 3, then 1 group of 3; or 3 groups of 4 then 1 group of 4; or 4 groups of 3.

5..Suppose you have 3 coins again, and one of them is fake. This time, you know that the fake coin has a different weight than the real coins, but you don't know whether it is lighter or heavier. How many trials do you need to find the fake coin?

Solution: 2 tries. Take a first coin and compare it with a second coin. If they are equal, you know the third is the fake. If they are not the same, compare the first coin with the third coin. If these two

are the same, you know the second coin is the fake. If they are not equal, you know your first coin is the fake.

6. Now you have 6 coins and two of them are fake. The fake coins are lighter than the real coins. Can you find both fake coins in 3 trials?

Solution: Yes, split the coins into two groups of three.

Case 1: The two groups weigh the same. You then know each group has one fake coin. Now use the same method in question 1 to find which coin is fake, for each group. That is 3 tries.

Case 2: The two groups weigh differently. Take the lighter group, and weigh two coins. If they are the same, they are your fake. If they are different, then the heavier one is the only real one.

CHALLENGE:

1. a. Now you have 12 coins. One of them is fake and has a different weight than the rest, but you don't know if it's heavier or lighter.

b. Can you determine if the coin is heavier or lighter than the rest in just three trials?

2. Suppose you have 16 coins. One of them is fake and has a different weight than the rest, but you don't know if it's heavier or lighter. Can you find the fake coin in 4 trials?