Roots of Unity – Problem Set 2

1. Let $\zeta = e^{2\pi i/5}$. Note that $\zeta^5 = 1$

   (a) Let $x = \zeta + \overline{\zeta}$. Compute and simplify the quantity $x^2 + x$.

   (b) Find the exact value of $\cos\left(\frac{2\pi}{5}\right) = \cos(72^\circ)$

2. Compute the area of the polygon whose vertices are the solutions in the complex plane to $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$. Express the area in simplest form as $\frac{a\sqrt{b}+c}{d}$.

3. Let $\omega = e^{2\pi i/3} = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$, and note that $\omega^2 = \overline{\omega}$ and $\omega^3 = 1$. Multiply out the expression

$$(x+y+z)((x+\omega y+\overline{\omega}z)(x+\overline{\omega}y+\omega z)$$

4. Let $y = \sin(2^\circ) + \sin(4^\circ) + \sin(6^\circ) + \cdots + \sin(178^\circ)$. Compute y and express your answer in terms of trigonometric functions of 1°.

Please work the problems given above as completely as you can. If you have any time left over, you may begin considering the following two problems, which I will solve in the remaining portion of my talk:

- Is it possible to load two six-sided dice in such a way that when you roll both of them, each of the numbers $2, 3, \ldots, 12$ is equally likely to occur as the sum of the two dice?
- [Putnam 2003 B5]: Let A, B, and C be equidistant points on the circumference of a circle of unit radius centered at O, and let P be any point in the circle's interior. Let a, b, c be the distance from P to A, B, C, respectively. Show that there is a triangle with side lengths a, b, c, and that the area of this triangle depends only on the distance from P to O.