

ROOTS OF UNITY – PROBLEM SET 1

1. Suppose $z = \frac{(2-i)^2}{(3+2i)^2}$. Find $\operatorname{Re} z$, $\operatorname{Im} z$, and $|z|$.
2. Find all complex numbers z such that $|z| = 1$ and $|z+1| = 1$. If z is any such number, what can be said about z^3 ?
3. Find all complex numbers z such that $z^6 + 7z^3 - 8 = 0$ and find the area of the smallest convex polygon that contains all of these numbers.
4. Identify the set of all complex numbers z such that $\left| \frac{z - \frac{1}{2}}{1 - \frac{1}{2}z} \right| = 1$.