Week 2 solutions!

Deven Ware, Intermediate Circle, Fall 2013

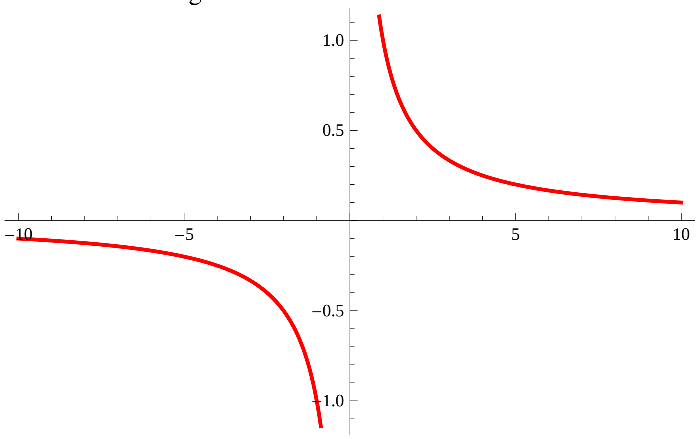
Warm up

- Mathematically forced, there were 6 pigeons (picks) and 5 holes (colors)
- Mathematically forced, (this depends on the class, but we have over 25 total) there are 25+ pigeons (people in the class) and 12 holes (months)
- Mathematically forced, there are 24 hours in a day so any two 15 hour sets must overlap.
- Strange coincidence, there are more holes (colors) then pigeons (instructors)

Problems

1. (a) He's wrong. A terminating decimal is indeed eventually periodic, it ends with an infinite string of 0's.
   (b) He's wrong. While it is true that after using up all 10 digits we have to use a digit that has already been used, this doesn't mean the decimal must repeat.
   (c) Here are a couple of examples, but there are many more: $.10100100010000\cdots$, $.123456789101112131415\cdots$
2. (a) $ad = bc$, to see this multiply both sides by bd.
   (b) same as (a)
3. (a) $.5$ the period is 0 with length 1
   (b) $.333\cdots$ the period is 3 with length 1
   (c) $.25$ the period is 0 with length 1
   (d) $.2$ the period is 0 with length 1
   (e) $.1666666\cdots$ the period is 6 with length 1
   (f) $.142857\cdots$ the period is 142857 with length 6
   (g) $.125$ the period is 0 with length 1
   (h) $.111\cdots$ the period is 1 with length 1
4. You probably shouldn't know how to do this at this point... its mostly to make you think. But the point is that we wound up with the same remainder and the remainder completely determines this. At this point I would accept that you got tired... or wound up starting to get the same numbers again.

5. It looks something like this:



6.

$$\begin{aligned} 60 &= \mathbf{8} \cdot 7 + 4 \\ 40 &= \mathbf{5} \cdot 7 + 5 \\ 50 &= \mathbf{7} \cdot 7 + 1 \\ 10 &= \mathbf{1} \cdot 7 + 3 \end{aligned}$$

7.

$$\begin{aligned} 130 &= \mathbf{8} \cdot 15 + 10 \\ 100 &= \mathbf{6} \cdot 15 + 10 \end{aligned}$$

Since the remainder repeated we're done, and we have $.866666\cdots$.

8.

$$\begin{aligned} 30 &= \mathbf{1} \cdot 22 + 8 \\ 80 &= \mathbf{3} \cdot 22 + 14 \\ 140 &= \mathbf{6} \cdot 22 + 8 \end{aligned}$$

Since the remainder repeated we're done, and we have $.136363636\cdots$

9. (a) The next step in the process is completely determined by the remainder from the previous step. Since there are only finitely many remainders we must eventually repeated remainders and hence repeat decimals.
   (b) 16
   (c) $n - 1$
   (d) there is nothing to do here

10. we saw before that the $\frac{1}{7} = .142857\cdots$. This has a period of length 6, so we just have to check where 2013 is in terms of multiples of 6. This is easiest with modular arithmetic notation, so I will write it that way. Now, we calculate $2013 = 213 = 33 = 3 \pmod 6$. Therefore we can see that the 2013$^{\text{th}}$ digit is the same as the third digit or 2.

11. Derek calculated this on the board in class and I won't actually write out the long division here because its terribly long... but

$$\frac{1}{17} = .0588235294117647\cdots$$

which has a period of length 16. Now there are two ways to interpret this question, where you'll get two different answers. When I was originally writing this problem, I for some reason had in mind digit being synonymous with significant figure and started counting at the 5. This gives you 0 as the answer (because you have to count 16 digits from the 5 to get a zero)

The smarter way, which I know agree with and which most people in class came up with was to think of digit as simply a place marker, so you start counting at the 0. This gives you 7 as the answer (because you simply count 16 digits from the start.)

Homework

1. (a) We just have to figure out what the denominator is in simplest form. 5^{4^2} cancels with the top, as does $8^{9^{10}}$ leaving just 3^2 or 9, so the maximum possible period length is in fact 1, but this takes some argument, from what we have proven here we can say 8.

   (b) Same idea as before, the denominator becomes 12 on simplification, and hence as we have proven the maximal period length is 11. But in this case the fraction actually becomes $\frac{1}{12}$ and you can calculate that decimal if you want to get a period length of 1 (its $.083333\cdots$)

2. (a) $\frac{4}{7} = .571428\cdots$(to see this multiply $\frac{1}{7}$ by 4) we have already calculated $2013 \pmod 6$ to be 3 so we can see that the answer is 1.

   (b) $\frac{503}{2000}$... there are a couple of ways to approach this, one is to simply calculate the decimal (its actually not hard, but I won't do it) another is to say by our argument it has to repeat after at most 1999 decimal places, but once it repeats itself is has to simply be 0, because 2000 has only prime factors 2 and 5, thus the 2013^{th} spot is 0.