Meeting 1 solutions!

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Warm up

- 4
- 16
- 2^{3^4}, since $4^{3^2} = 2^{2(3^2)} = 2^{18}$ and $2^{3^4} = 2^{81}$.
- By the pigeon hole principle. There are three numbers and each is either non-negative or negative. Thus we have two of the same sign, and the product of two numbers with the same sign is positive.
- $2013 = 3 \cdot 11 \cdot 61$ and hence we can have all possible products of these three numbers, yielding 8 total. (because we have to remember to include 1!)

Problems

1. (a) 70
   (b) 200
   (c) $n(n+1)$
   (d) n^3
2. (a) convert to a common denominator using the LCM, this is $\frac{21}{70}$ and $\frac{25}{70}$ so you should take the deal.
   (b) this is $\frac{43}{44}$ vs $\frac{43}{45}$, of course you should reject the deal.
   (c) again we use a common denominator. $\frac{9}{12} = \frac{45}{60}$, $\frac{11}{15} = \frac{44}{60}$ and $\frac{15}{20} = \frac{45}{60}$. So as long as you don't choose to trade for $\frac{11}{15}$ you are fine here. According to William the ideal solution is to steal you friends watermelon.
3. There are 72 pedals, which tells us we have 36 total people. If they all rode unicycles this would yield 36 wheels. Since there are 76 wheels this means we need to find 40 extra wheels, hence 20 people ride tricycles.
4. (a) Nope, try $\frac{1}{1000001}$ and anything else.
   (b) yep! take any common denominator and multiply it by a million!
   (c) yep! take any common denominator and multiply it by 2
   (d) no, try to do this with $\frac{1}{2}$ and anything else ...
   (e) no, try to do this with $\frac{1}{3}$ and anything else ...
   (f) yep! take any common denominator and square it!
5. (a) Theres no relation. Try $\frac{a}{b} = \frac{1}{2}$ and $\frac{c}{d} = \frac{50}{99}$ then clearly $cd > ab$, but taking $\frac{a}{b} = \frac{500}{1000}$ and $\frac{c}{d} = \frac{50}{99}$ has $ab > cd$.
   (b) Theres no relation. Try $\frac{a}{b} = \frac{1}{2} < \frac{1}{1} = \frac{c}{d}$ then $ac < bd$, but trying $\frac{a}{b} = \frac{1}{2} < \frac{50}{1} = \frac{c}{d}$ gives us $ac > bd$.
   (c) $ad < bc$, multiply both sides of the equation by bd to see this.

(d) this depends on how you interpret it. If we force a and c to carry the negative sign then everything stays the same. if we let the negative sign go anywhere, we lose all relations.

6. The rule is: go to the first digit where the two decimals differ, whichever one is larger there is the bigger number.

7. (a) $\frac{12345}{1000} = 12.345 < 14.2 = \frac{142}{10}$
   (b) $\frac{3333}{500} = 6.666 < 6.667 = \frac{6667}{1000}$
   (c) $\frac{36}{5} = 7.2 > 7 = \frac{70}{10} < \frac{135}{20} = 6.75 = \frac{27}{4}$

8. (a) $\frac{3}{7} = .428571\cdots$
   (b) $\frac{1}{6} = .16666\cdots$
   (c) $\frac{7}{9} = .7777\cdots$

9. I certainly can't. We'll prove its impossible eventually.

10. (a) The only primes in the denominators are 2 and 5.
    (b) $\frac{1}{10^n} = .00\cdots01$ with $n-1$ zeroes, so this works.
    (c)
$$\frac{a}{2^k5^{k+n}} = \frac{2^na}{2^n2^k5^{k+n}} = \frac{2^na}{10^{k+n}}$$
    which terminates because it has a denominator of the form 10^{k+n}.
    (d) this is the same as the previous part, just switch the role of 2 and 5.
    (e) If a fraction has a denominator with only primes 2 and 5, then it has a denominator which looks like one of the previous three parts of this problem, and hence it terminates.

Homework

1. (a) 200
   (b) These are all primes, so their least common multiple is just their product.

2. (a) the powers of three and seven in the denominator cancel with those in the numerator and hence the only primes in the denominator are 2 and 5, so the decimal terminates by problem 10.
   (b) $2013 = 3 \cdot 11 \cdot 61$ and $2012 = 2^2 \cdot 503$. Therefore we cancel the $503, 11, 61, 3$ out of the denominator and the only thing in the denominator is powers of 5, so again by problem 10 this terminates.