

FROM FRACTIONS TO DECIMALS

LAMC INTERMEDIATE GROUP - 9/29/2013

WARM UP! (SINCE THE SUMMER HAS BEEN SO COLD...)

- $2^2 =$
 - $2^{2^2} =$
 - Which is bigger 2^{3^4} or 4^{3^2} ?
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- Show that the product of some two of the following numbers is non-negative
(*Hint: Don't try to calculate these crazy numbers!*)

$$\frac{65523246}{13580164} - 4.8249, \quad 1.015 - \frac{3e^{\log \pi}}{\pi^3}, \quad \frac{2^{2^{2^2}}}{8^{50}} - 1.45$$

- How many factors does 2013 have? (*Hint: list out all possible factors, beginning with the primes*)

(1) What is the least common multiple of the following sets of positive integers?

- 10, 14

- 8, 50

- $n, (n + 1)$

- n, n^2, n^3

(2) Your very sneaky friend really likes fruits, and as such he has offered you a sequence of trades, but he offered them in a deceiving way. For each of the following, decide whether or not you should take his offer (with explanation!) (Note: In each part of the problem, assume you and your friend started with identical objects)

- He'll give you $\frac{3}{10}$ of his mango for $\frac{5}{14}$ of yours.

- He'll give you $\frac{43}{44}$ of a banana for $\frac{43}{3}$ of $\frac{1}{15}$ of your banana.

- Finally, he has offered to give you $\frac{9}{12}$ of his watermelon for *either* $\frac{11}{15}$ or $\frac{15}{20}$ of yours. What should you do?

(3) In a very unusual town everyone rides either a tricycle or a unicycle. In total for the town there are 72 pedals and 76 wheels, how many people ride tricycles?

(4) Given any two fractions, can you always find a common denominator that is...

- smaller than 1,000,000?

- bigger than 1,000,000?

- even?

- odd?

- a power of two?

- a square?

(5) Suppose you have two fractions

$$\frac{a}{b} < \frac{c}{d}$$

($a, b, c, d > 0$). What (if anything) can you tell about the relation ($<$, $>$, $=$) between...

- ab and cd ?

- ac and bd ?

- ad and bc ?

In each case, if there is a relation prove it. If not, give examples of fractions where all three possibilities ($<$, $>$, $=$) can hold.

- **(CHALLENGE)** What (if anything) changes if we allow the fractions to be negative?

(6) It is often easier to compare decimals than fractions, for instance it is easier to compare .16 and .15625 than $\frac{4}{25}$ and $\frac{5}{32}$. Come up with a formulaic rule that is consistent with this.

(7) Given an arbitrary fraction, we can always convert it to a decimal by performing long division (because of course, we can't forget that $\frac{a}{b}$ means a divided by b). Use long division to convert the following fractions to decimals and put them in increasing order:

- $\frac{12345}{1000}, \frac{142}{10}$

- $\frac{6667}{1000}, \frac{3333}{500}$

- $\frac{36}{5}, \frac{27}{4}, \frac{70}{10}, \frac{135}{20}, 7$

- (8) Notice that the decimals in the previous problem always terminated, but we know that isn't always the case! Find the decimal equivalent of the following fractions:

• $\frac{3}{7}$

• $\frac{1}{6}$

• $\frac{7}{9}$

(9) Now we have looked at two different sets of fractions: in the first group all the decimal equivalents terminated, in the second they all eventually repeated themselves. Can you come up with a fraction so that the decimal equivalent does neither of these?

(10) **(CHALLENGE)** The denominators in problem 6 all had a special property which made the decimal equivalent terminate. Figure out what this property is, in this problem, we figure out what that property is and attempt to prove that the decimals always terminate.

- What is the property of the denominators in problem 6? (*Hint: Here are some more denominators with this property: 50, 2^{2^2} , $5^{2^{2^2}}$, 125, 20, 40*)

- Prove that if a fraction has denominator 10^n then the decimal equivalent terminates (*Hint: write the fraction as $a \cdot \frac{1}{10^n}$, what is the decimal equivalent of $\frac{1}{10^n}$?*)

HOMEWORK PROBLEMS

(1) What is the least common multiple of the following sets of positive integers?

- 4,8,50

- 19,29,61 (*your answer can be expressed as a product of primes*)

(2) Prove that the decimal equivalent of the following fractions terminate:

- $\frac{3^4 7^2 2^3}{3^2 7^2 5^2 2^2}$

- $\frac{2013 \cdot 2012}{503 \cdot 11 \cdot 61 \cdot 3 \cdot 5^4}$