

LAMC Junior Circle

June 9, 2013

Please print your name, first and last, followed by your school grade in the space below.

First Name

Last Name

Grade

The year-end problem solving session

Pr #	1	2	3	4	5	6
Score	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$

7	8	9	10	Total
$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{100}$

Problem 1

a. Solve the following equation.

5 pts

$$x^2 \equiv 2 \pmod{7}$$

In other words, find all the congruence classes x among $0 \pmod{7}$, $1 \pmod{7}$, $2 \pmod{7}$, $3 \pmod{7}$, $4 \pmod{7}$, $5 \pmod{7}$, and $6 \pmod{7}$ such that $x \times x \equiv 2 \pmod{7}$.

b.

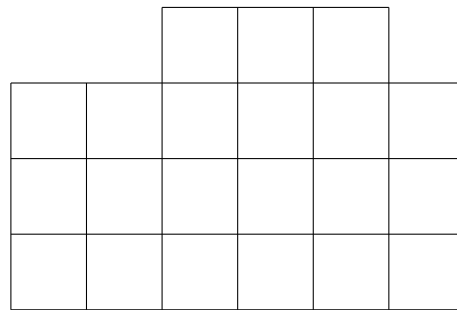
5 pts

$$\sqrt[3]{3} \equiv \quad \pmod{5}$$

Problem 2

10 pts

Trace the lines that would cut the following figure into three parts of equal shape and size.



Problem 3

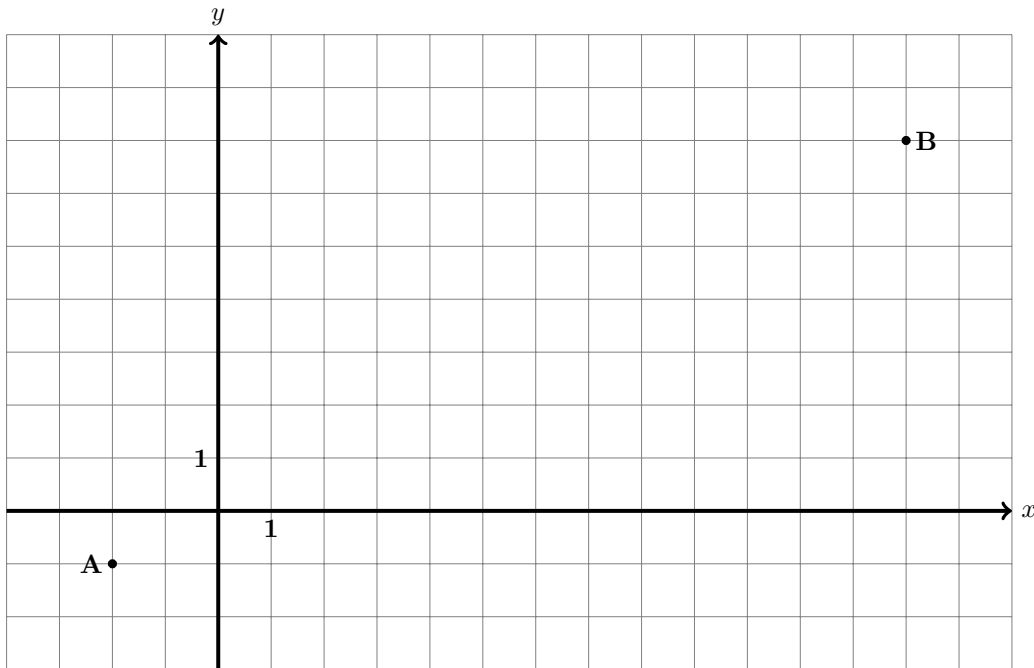
10 pts

Find the number that is seven times greater than its last digit.

Problem 4

10 pts

The distance $|AB|$ is a whole number. Use the Pythagoras' theorem to find it.



$$|AB| =$$

Problem 5**10 pts**

Write down the first six lines of the Pascal's triangle in the space below.

Problem 6**10 pts**

Use the previous problem to expand the following product.

$$(x + y)^5 =$$

Problem 7**10 pts**

You toss a fair coin five times. How likely are you to get either two heads or two tails? Hint: you may find it helpful to use either Problem 5 or Problem 6.

$$P(\text{two heads or two tails}) =$$

Problem 8**10 pts**

There are 23 students, 3 of them identical (indistinguishable) triplets, in a class. How many ways are there to put the class in one line for taking pictures in such a way that the triplets are in the middle of the line? Answering this question, do not compute the factorials!

Problem 9

10 pts

Draw a 4D cube (a.k.a. a hypercube and a tesseract) in the space below. How many 3D faces does it have?

The number of the 3D faces =

Problem 10**10 pts**

Hiking in Yosemite, Oleg used a rectangular soap bar to wash the dishes, spending the same amount of soap each time. After seven washes, the length, width, and height of the soap bar became two times smaller than the original sizes. How many more times can Oleg wash the dishes with the soap?