

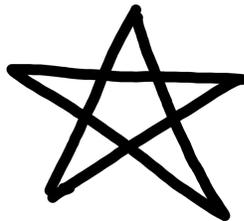
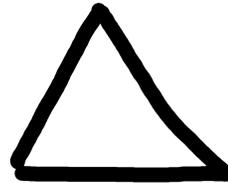
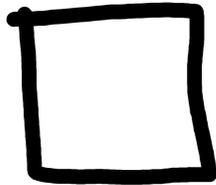
GEOMETRY 3

BEGINNER CIRCLE 5/19/2013

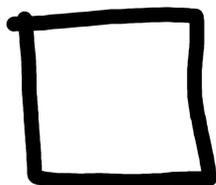
1. COUNTING SYMMETRIES

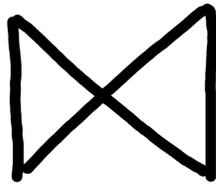
A drawing has symmetry if there is a way of moving the paper in such a way that the picture looks after moving the paper.

Problem 1 (Rotational Symmetry). A shape has rotational symmetry if you can rotate the paper around a point to return the shape to itself. The degree of symmetry around the point is the number of different rotations around that point that return the shape to itself. For the following shapes, mark their point of rotational symmetry, and the degrees of symmetries they have.



Problem 2 (Reflexive Symmetry). A shape has reflexive symmetry if you can reflect the shape across a line and return the shape to itself. This means that there is a line where the picture is a mirror image of itself across the line. For the following shapes, mark the lines of reflexive symmetry.



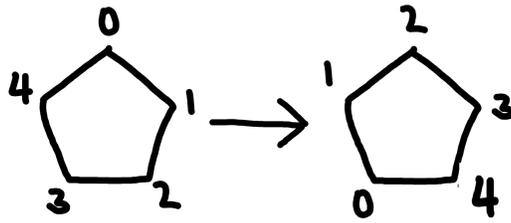


Problem 3. Draw a picture that has Reflexive symmetry, but not rotational symmetry

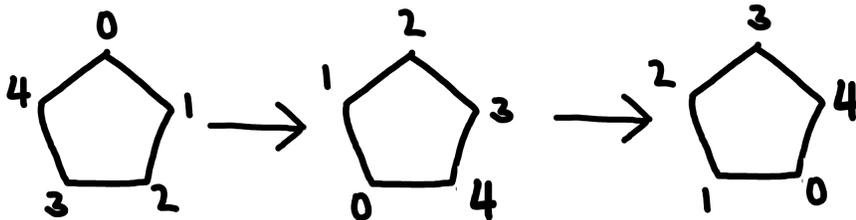
Problem 4. Draw a picture that has rotational symmetry, but no reflexive symmetry

Problem 5. Let us come up with a way of writing down rotational symmetries. When I have a picture with rotational symmetries labeled, I write R_n to mean the rotation that takes the point labeled 1 to the point labeled n . For instance, the rotation R_3 is

drawn below.



We can condense two rotations into a new rotation. For example, $R_3 \cdot R_4$ means rotate the point labeled 0 to the point labeled 3, then rotate 4 more times clockwise.



However, when we do 2 rotations, we get a new rotation! Find out what the composition of these rotations are

(1) $R_2 \cdot R_2$

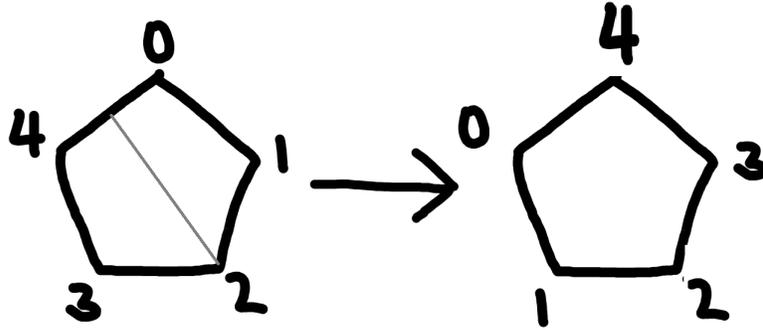
(2) $R_3 \cdot R_2$

(3) $R_4 \cdot R_3$

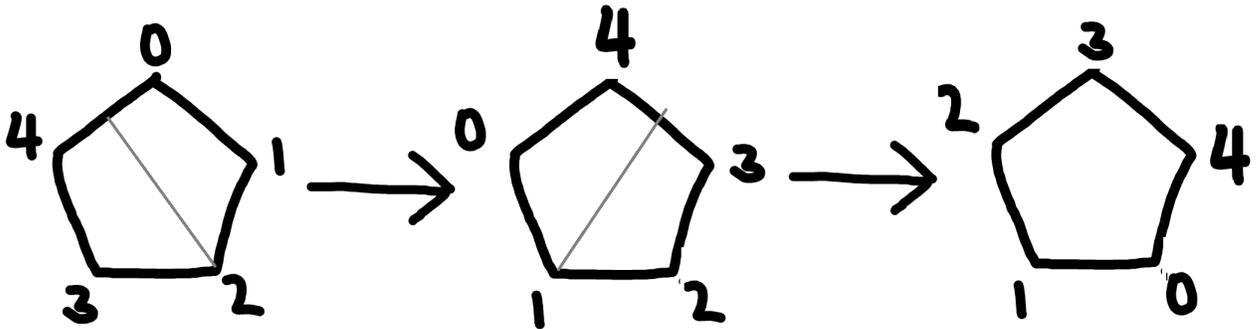
(4) $R_0 \cdot R_3$

Problem 6. Similarly, we can come up with a way for writing down reflections. When I write F_n , (f for reFlection), I mean *reflect across the line going through the center and*

the point n . For example, the reflection F_2 does the following to the picture



What happens if we do two reflections? (I've given an example below of this below of $F_2 \cdot F_3$. Be careful! The reflection F_3 is always the reflection that goes through the vertex *originally* labeled 3!!



Problem 7. Express the following compositions of reflections as rotations

(1) $F_2 \cdot F_4$

(2) $F_2 \cdot F_2$

$$(3) F_3 \cdot F_1$$

Problem 8. Clearly, we can put reflections and rotations together

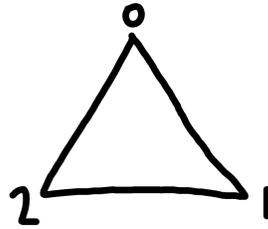
$$(1) R_2 \cdot F_2$$

$$(2) F_2 \cdot R_2$$

$$(3) F_3 \cdot R_3$$

$$(4) F_3 \cdot F_2 \cdot R_2$$

Problem 9 (Multiplication tables). A multiplication table tells you how the various symmetries interact with each other. Fill out the multiplication table for the below shape.



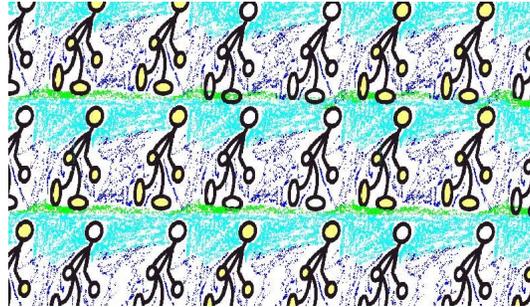
I've filled in a couple of spaces to help. Notice that the multiplication goes *top* of the table, then *side*. (If you are not sure what this means, ask an assistant!)

	R_0	R_1	R_2	F_0	F_1	F_2
R_0				F_0		
R_1		R_2				
R_2						
F_0						
F_1					R_0	
F_2				R_1		

Problem 10. Explain why if a shape has 2 different kinds of reflexive symmetry, it automatically has rotational symmetry.

2. WALLPAPER

We now add in a third type of symmetry: translation. A picture is translation invariant if you can move the picture around without rotating or reflecting and get back to itself. For example, train tracks exhibit translational symmetry

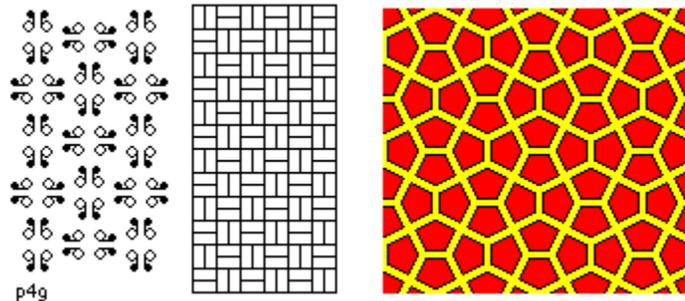


Finally, a last kind of symmetry, called “Glide reflection”. This symmetry is when you reflect and translate along the same line.



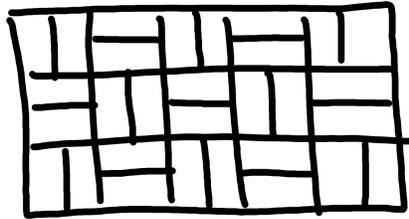
When you have a wallpaper, there is a section of the design where every other part of the design is some reflection or translation or rotation of that part. This section is called the “Fundamental Domain”

For each of the following wallpapers, identify the rotational, glide and reflexive symmetry involved. I’ve done an example for you on the next page.

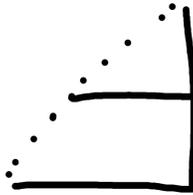


Problem 11. Fill out the following information

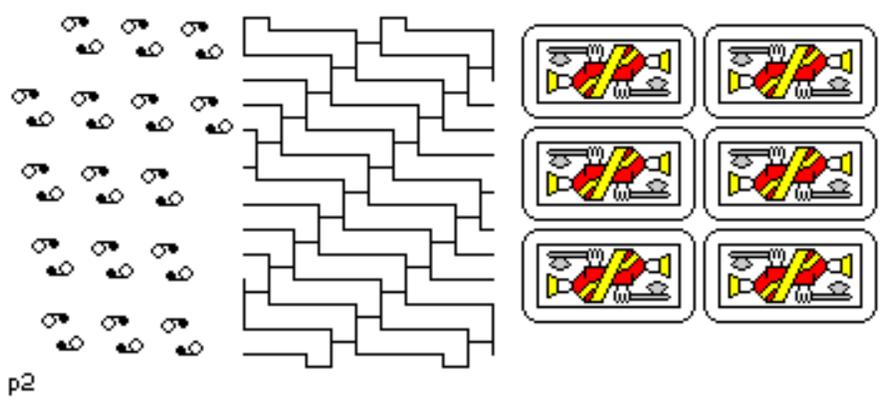
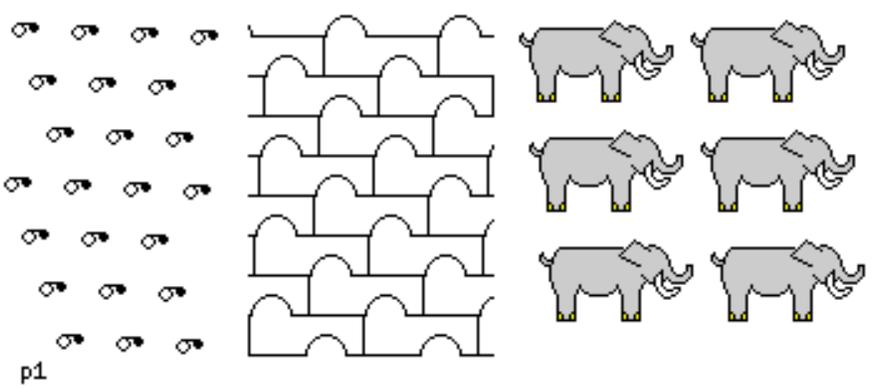
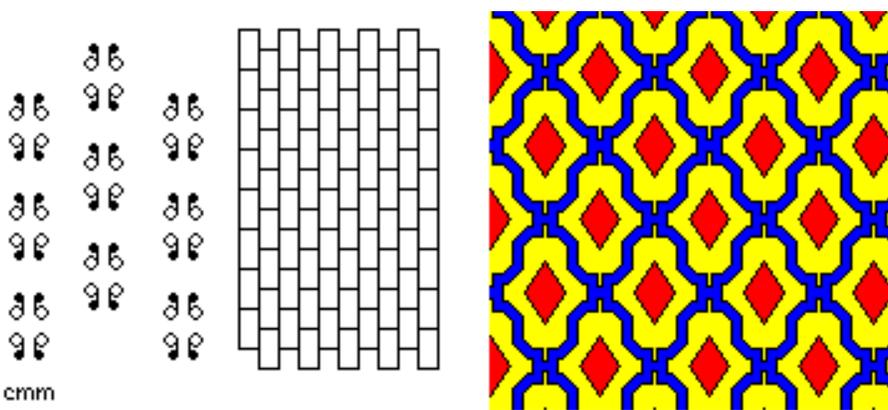
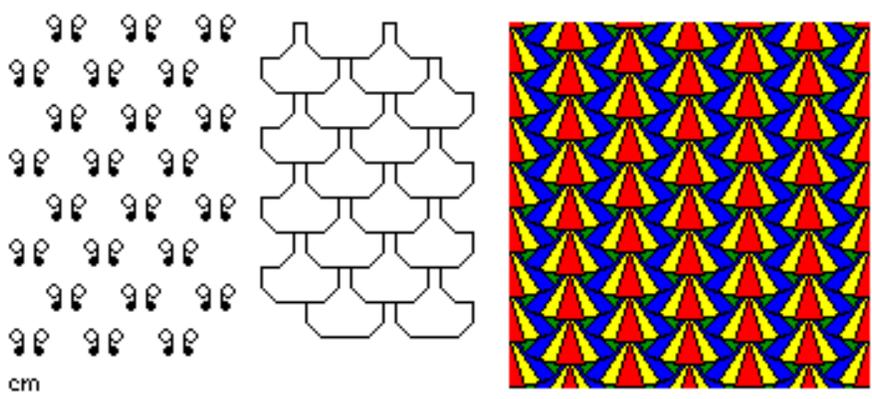
- Symmetry Name: P4G
- Draw the wallpaper

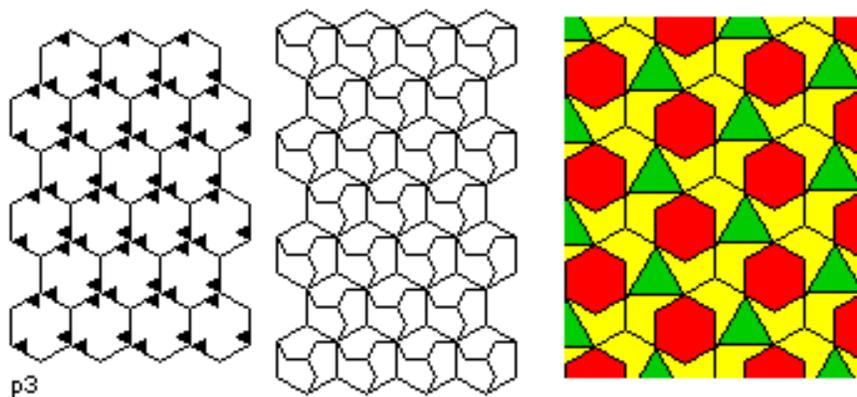


- Draw the Fundamental Domain of the pattern, and mark it on the wallpaper

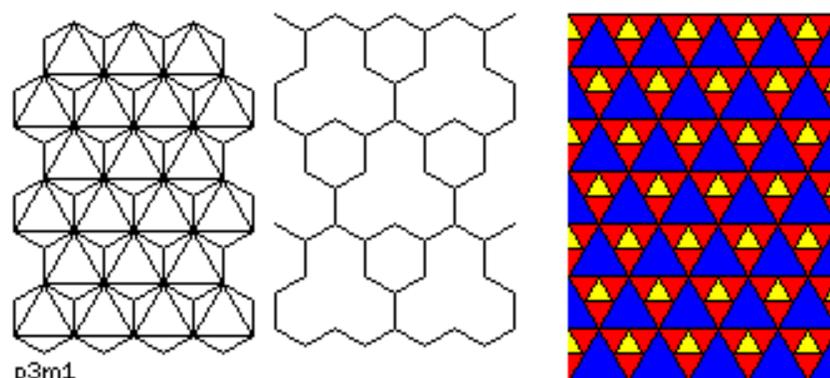


- How many lines of Reflection are there in the fundamental domain?
There are two lines of reflection, one horizontal and one vertical.
- Types of Rotational Symmetry (that do not lie on lines of reflection)
There is a 4 fold symmetry in the top right corner
- Types of Rotational Symmetry (that do lie on lines of reflection)
There is a 2 fold rotational symmetry where at the center of the design.
- Types of Glide Reflection Symmetry

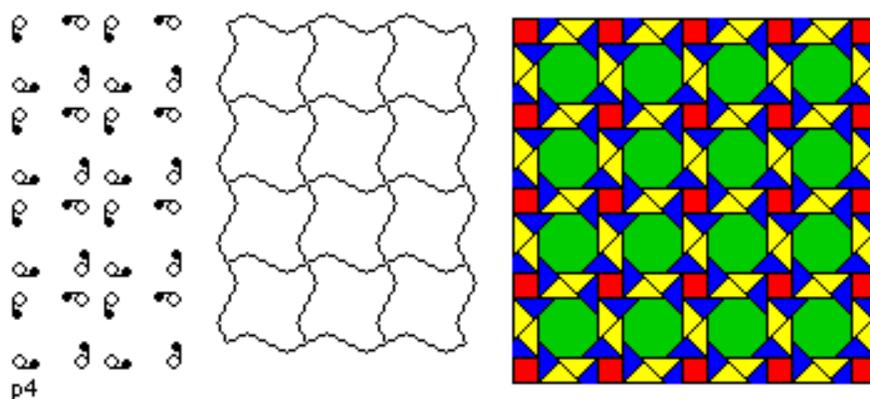




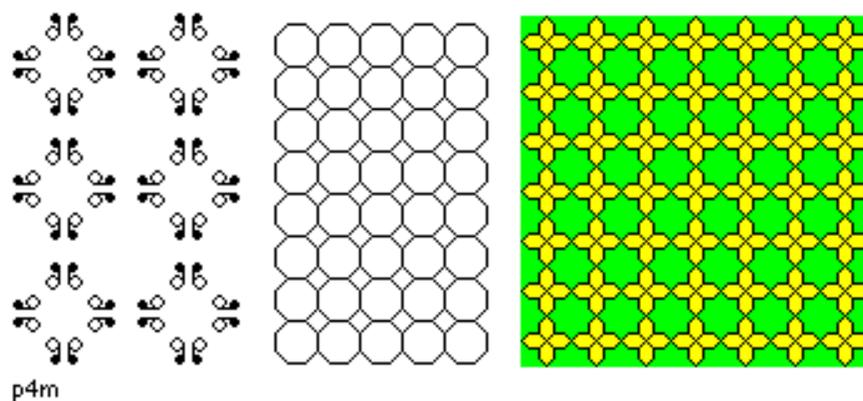
p3



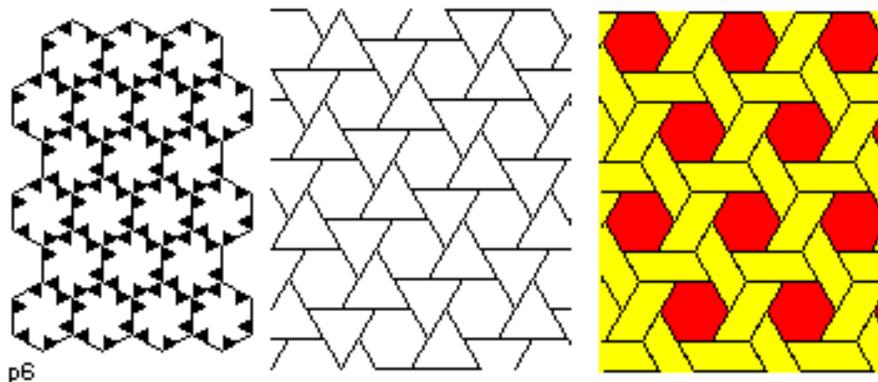
p3m1



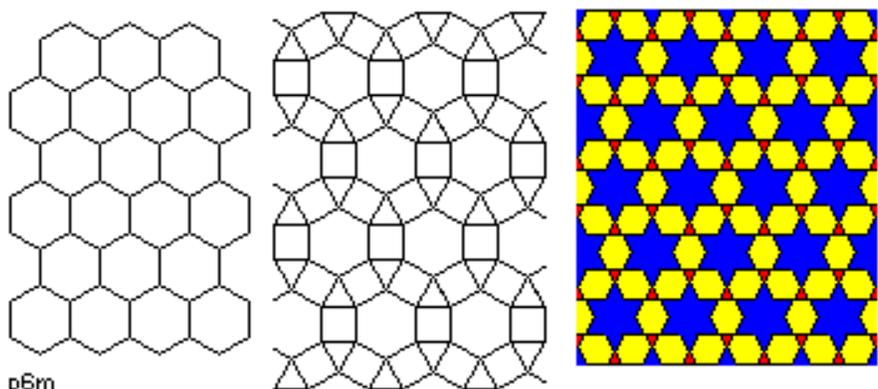
p4



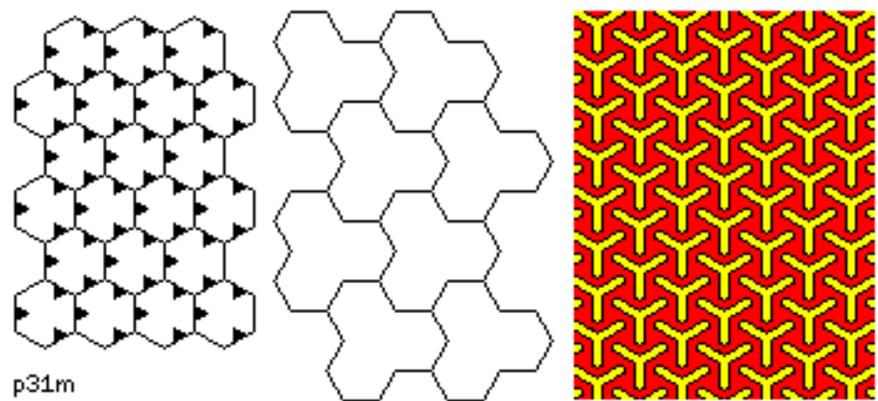
p4m



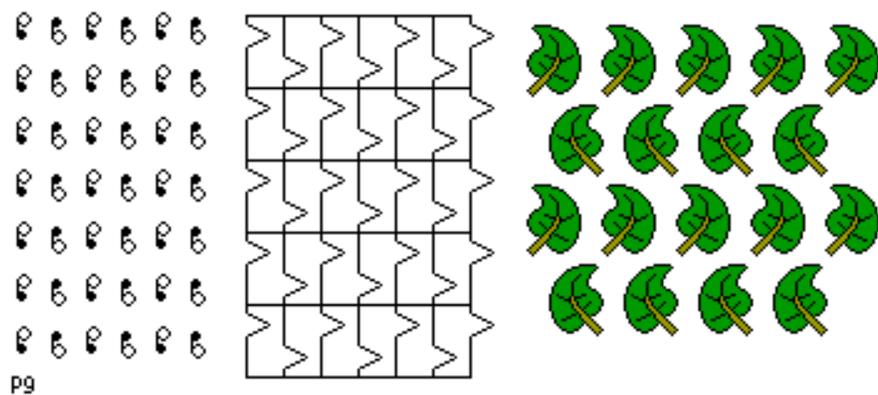
p6



p6m



p31m



p9

