LAMC Junior Circle **May 19, 2013**

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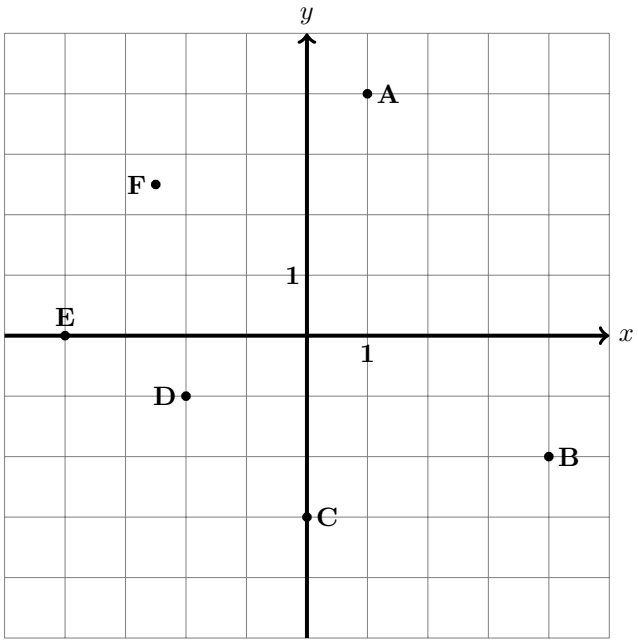
Problem 1 *The first day of a year is a Tuesday. The year is not a leap year. What day of the year is June 27th?*

Problem 2

$$2 \div 3 \equiv \qquad (mod\ 5)$$



Problem 3 *Write down the coordinates of the points on the picture above.*

$$A = (\quad , \quad) \qquad B = (\quad , \quad)$$

$$C = (\quad , \quad) \qquad D = (\quad , \quad)$$

$$E = (\quad , \quad) \qquad F = (\quad , \quad)$$

Problem 4 *A function is given by the following formula.*

$$y = 8x - x^2 \tag{1}$$

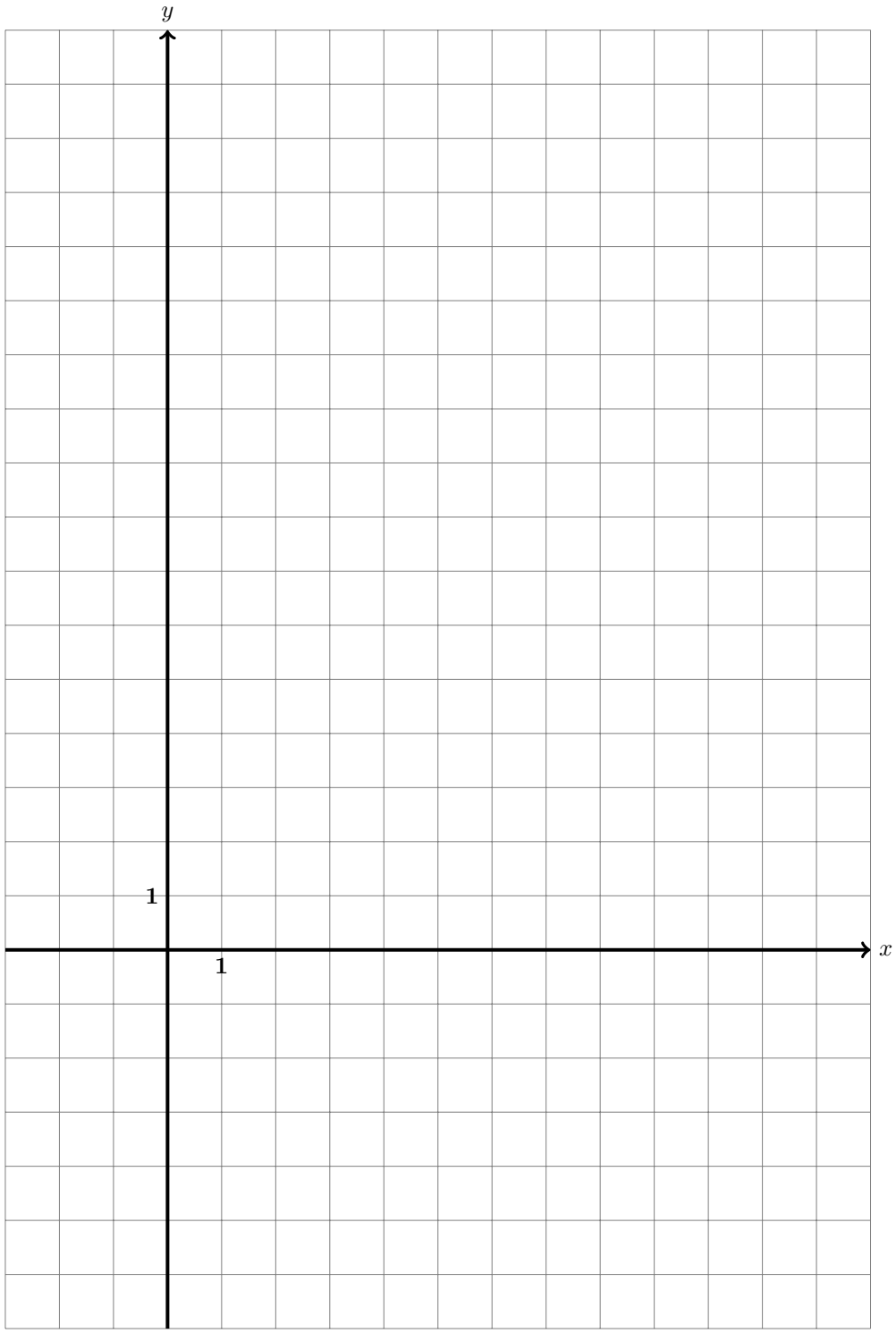
Fill out the table below.

x	-2	-1	0	1	2	3	4
y							

x	5	6	7	8	9	10	11
y							

- *Mark the points (x, y) from the above table on the coordinate plane below. Some of the points will not fit, mark only those that do.*

- *Graph the function $y = 8x - x^2$ on the grid below.*


y
x
1
1

Definition 1 *A point x_m is called the global maximum of the function $y = f(x)$, if $f(x_m) > f(x)$ for any point $x \neq x_m$.*

In other words, the point $(x_m, f(x_m))$ is the highest point on the graph of the function.

Problem 5 *Find the global maximum of the function $y = 8x - x^2$. What is the value of the function at that point?*

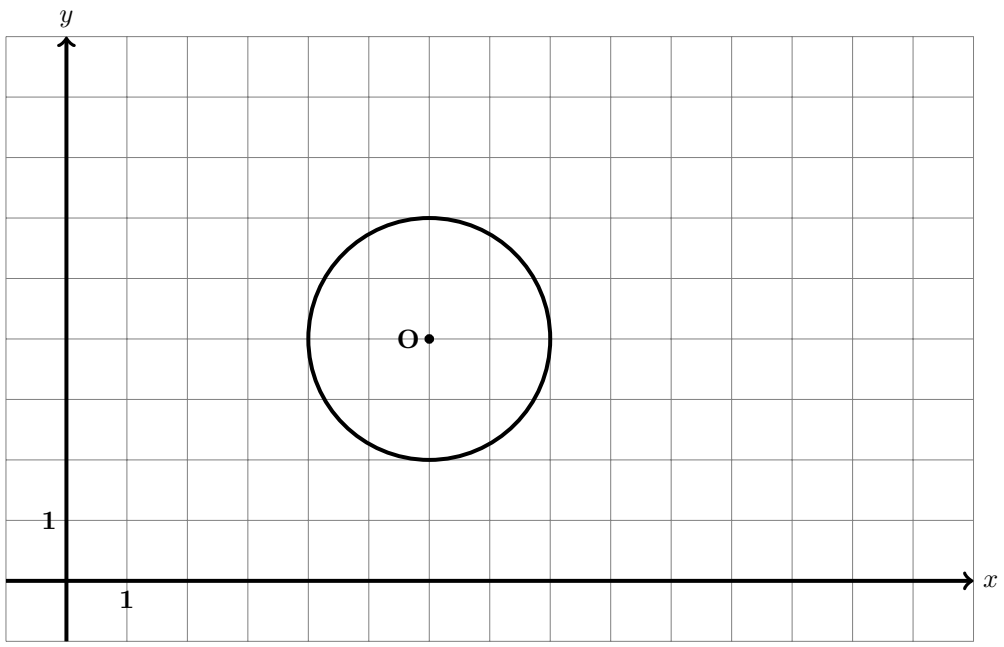
$$x_m = \qquad\qquad f(x_m) =$$

Problem 6 *What rectangle has the largest area out of all the rectangles with the 16" perimeter?*

Problem 7 *Use a compass and a ruler to construct a* 60° *angle in the space below.*

Problem 8 *Use the Pythagoras' Theorem to find* $|AB|$.

Problem 9 *Write down the equation of the following circumference.*

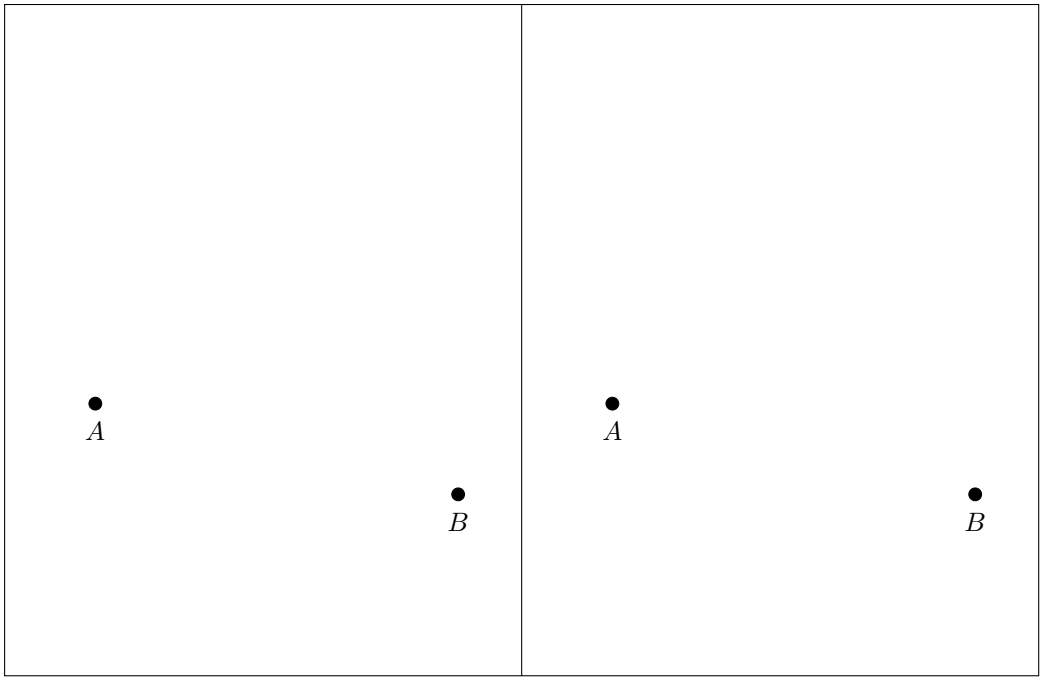


Does the point $A = (6, 1.99)$ *lie on the circumference? Why or why not?*

Problem 10 *How many different cubic roots of one are there in the mod* 7 *arithmetic?*

Problem 11 *Prove that the shortest path between any two different points in the Euclidean plane is the segment of the (unique) straight line passing through them.*

Problem 12 *Use a compass and a ruler to construct the shortest path connecting the points A and B on the right-hand side copy of the cylinder below.*



Problem 13 *Assuming that your favourite book has more than 27 words, prove that every 27-word sequence in there has at least two words that begin with the same letter.*

Problem 14 *You are given a standard (8×8) chess board with a pair of the diagonally opposite corner squares removed. You are also given a set of dominoes, each piece having the same width as the chess board square and twice its length. Is it possible to cover the board with the dominoes? Why or why not?*

Problem 15 *An astronaut has landed on an asteroid's equator. The asteroid has the shape of a ball. The astronaut went 10 miles North, then 10 miles East, then turned South and walked another 10 miles. As a result, he was 20 miles East of the point of his disembarkation. How many more miles has he go East to get back to the landing point?*

A 4-dimensional triangular pyramid is also known as a 4D simplex and a pentachoron.

Problem 16 *Draw a pentachoron in the space below. Hint: recall how you manage to draw a 3D triangular pyramid, a.k.a. a 3D simplex or a tetrahedron, on a 2D sheet of paper.*