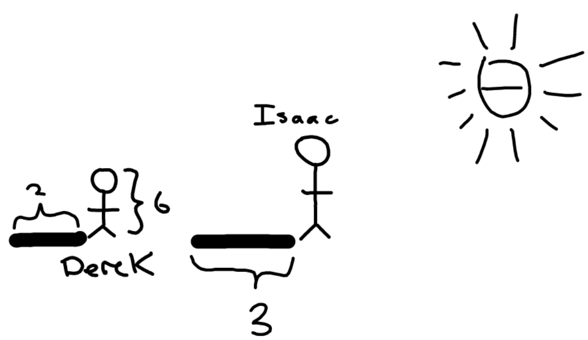
GEOMETRY 2

BEGINNER CIRCLE 5/12/2013

1. Shadows and things about Triangles.

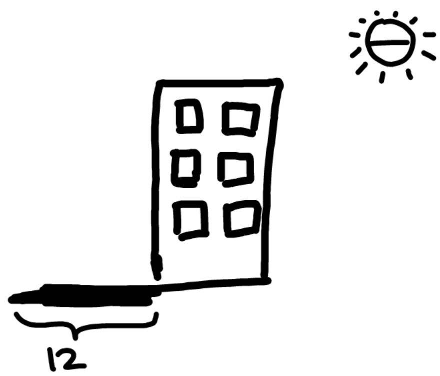
Isaac and Derek are measuring the heights of various objects. They don't know what time it is, but they notice that Derek is 6 feet tall, and has a shadow of 2 feet in length.



Problem 1. Isaac looks at his shadow, and notices that it is 3 feet long. How tall is Isaac?

Jeff Hicks/Los Angeles Math Circle/UCLA Department of Mathematics .

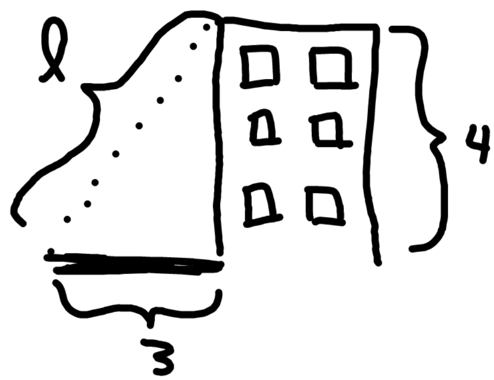
Problem 2. Isaac and Derek look at a nearby building, and notice that the shadow of the building is 12 feet long. How tall is the building?



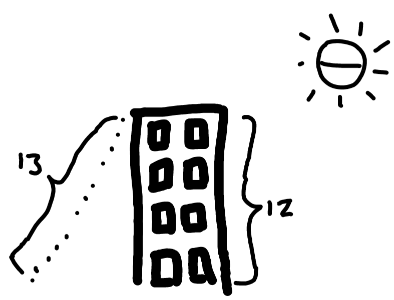
Problem 3. Isaac hates the sun, so he desires to stand in the shadow of the tall building. How far away from the building can Isaac stand and still be in the shade?

Problem 4. Derek loves to be in the sun, and tries to stay in the sun as much as possible. As a result, Derek uses a ladder to get from the bottom of the building to the top of the building. His requirement for placing the ladder is that the entirety of the ladder is in the sun. He knows for a building with height h,and shadow length s, he needs a ladder of length $l = \sqrt{h^2 + s^2}$ in order for the ladder to be completely outside of the shadow.
Suppose that Derek wants to climb a building of height 4, and has a shadow of length

3. How long of a ladder does he need?

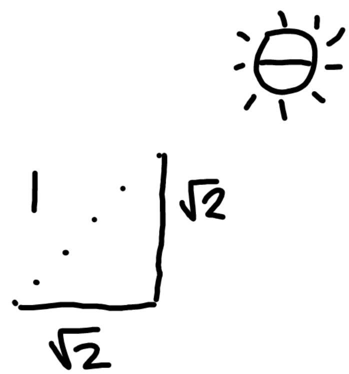


Problem 5. Derek wants to scale a different building, and needs a ladder of length 13 to do so. The building that he wants to climb has a height of 12. How long is the shadow?

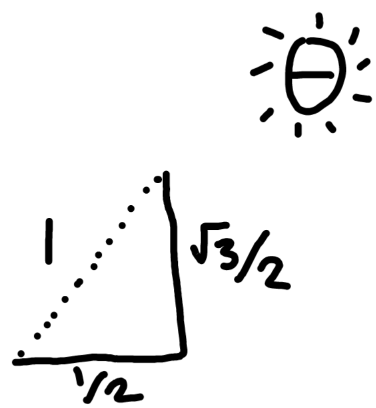


Problem 6. Neither Derek or Isaac have watches, so they need a different way to figure out what time of day it is. Derek method for classifying the "time" of day is to look at the tallest building that he can climb with a 1 ft long ladder, while still staying in the sun. For instance, if the sun is as pictured below, what time would Derek say

it was?

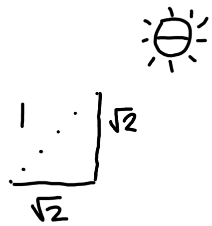


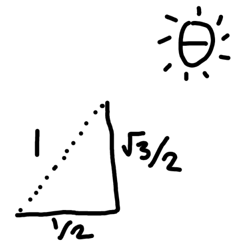
Problem 7. Isaac has a different method for classifying what time it is. He looks at the longest shadow that he can traverse with a 1 foot long ladder. If the sun was pictured as below, what time would Isaac say it was?



Problem 8. Jonathan has a 3rd method to classifying what time it is. Jonathan looks at a building, and says that the time is the height of the building, divided by the length of the shadow. Write down what time Derek, Isaac and Jonathan would say it

was for each of the following sun-positions.





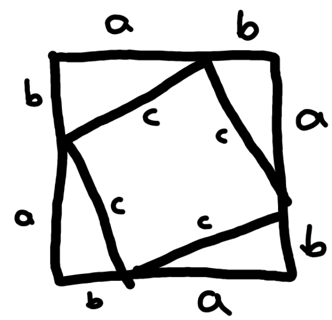
Problem 9. How can you take suppose that Derek says that it is $\frac{4}{5}$ and Isaac says that it is $\frac{3}{5}$. What time would Jonathan say it was?

Problem 10. Suppose that Isaac measures the time to be $\frac{12}{13}$. What would Derek measure the time to be?

Problem 11. Suppose that Derek measures the time to be $\frac{8}{17}$. What does Jonathan measure the time to be?

Problem 12. Show that if Isaac measures the time as $I(\theta)$, and Derek measures the time to be $D(\theta)$, that $I(\theta)^2 + D(\theta)^2 = 1$.

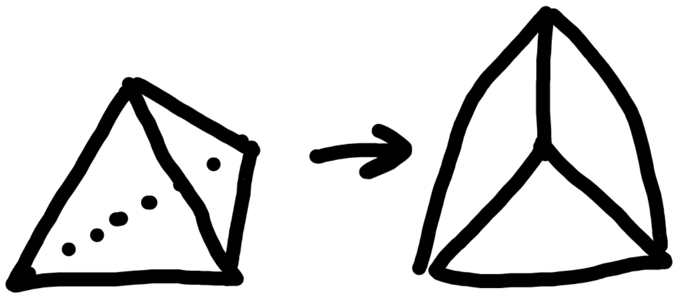
Problem 13. Compute the area of the square two different ways.



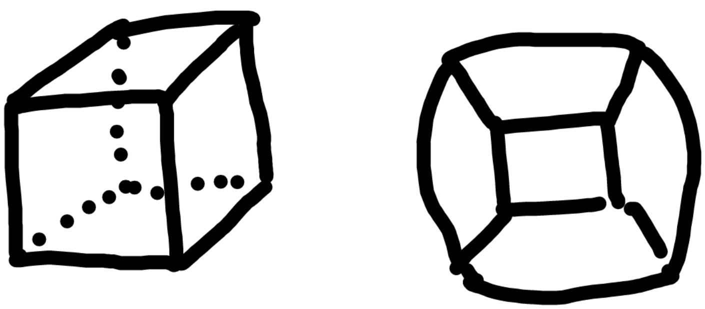
(a) Show that the area of the square below is $a^2 + b^2 + 2ab$.
(b) Show that the area of the square below is $c^2 + 2ab$.
(c) Conclude that $a^2 + b^2 = c^2$.

2. Platonic Solids

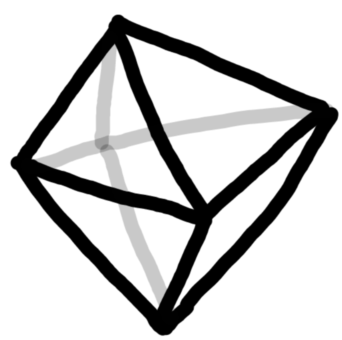
A platonic solid is a shape where all the faces have the same number of sides, and all of the faces are regular polygons. Sometimes, it is really hard to draw 3d shapes, you can just draw the shape flattened out. For example, this shape (a triangular pyramid, or tetrahedron) has a flattened out version that looks like this:



A cube, has a flattened out version that looks like this:



Problem 14. Can you draw a flattened out version of the octahedron? (looks like two stacked square pyramids)



Problem 15. For every platonic solid, let V be the number of vertices's (corners) the shape has, E be the number of edges, and let F be the number of faces.

(a) What is V, E and F for a tetrahedron?

(b) What is V, E and F for a cube?

(c) What is V, E and F for the octahedron?

There are two more platonic solids. Let's see if we can find them.

Problem 16. Verify for the above shapes, that $V - E + F = 2$

Problem 17. Each of the platonic solids is given by 2 numbers: the number of sides each face has, and the number of edges that meet at each vertex. Let us call these S and A. What is

(a) S and A for the tetrahedron?

(b) S and A for the cube?

(c) S and A for the octahedron?

Problem 18. If we know that S is (the number of sides of each face) and what F is, how can we find out what E is?

Problem 19. Likewise, if we know what A is (the number of edges per vertex) and V is, can you find out what E is?

Problem 20. Using the "fact" that $V - E + F = 2$, and the above equations , show that

$$\frac{2E}{S} - E + \frac{2E}{A} = 2$$

Problem 21. Manipulate the above equation to get

$$\frac{1}{A} + \frac{1}{S} = \frac{1}{E} + \frac{1}{2}$$

Problem 22. The number of sides must be greater than zero. Therefore, we have that $\frac{1}{A} + \frac{1}{S} > \frac{1}{2}$. What are all the different possible whole number values for A and S that satisfy the equation?

Problem 23. Draw a flattened shape where each of the faces has 5 sides, and each of the vertices has 3 edges

Problem 24. Draw a flattened shape where each of the faces have 3 sides, and each of the vertices has 5 edges.