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Theorem 1 *A positive integer is divisible by 11 if the difference between the sum of its odd-numbered digits and the sum of its even-numbered digits is either zero or is divisible by 11.*

Problem 1 *Use Theorem 1 to see if the following integers are divisible by 11. Each time, check you conclusion by means of a direct computation.*

a. 77

b. 1,518

c. 11,264

Problem 2 *(Problem 5 from the first handout)*

The number 1089 *has the following magical property. Take any three-digit number that has more hundreds than singles. Reverse the order of the digits and subtract the second number from the first. If you get a two-digit number as a result, write* 0 *in the third position from the right (first from the left) so that the difference is again a three-digit number. Reverse the order of the digits in the difference and add up the difference to the resulting number. The final sum will always be equal to* 1089*.*

Prove the magical property of the number 1089*.*

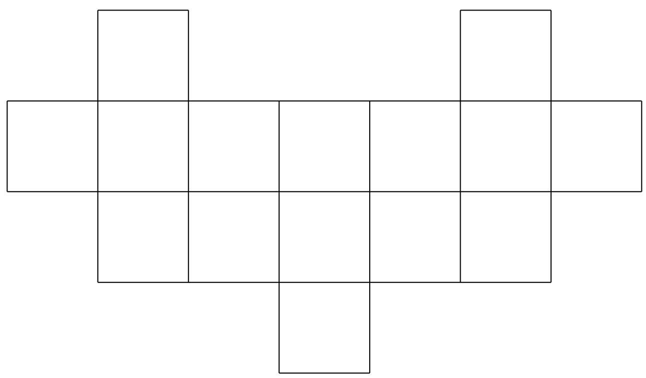
Problem 3 *(Problem 8 from the second handout)*
Does there exist a positive integer such that its last two digits are 11, it is divisible by 11, and the sum of its digits equals 11? Why or why not?

When this problem is solved, let us switch back to page 10 of the previous (third) handout.

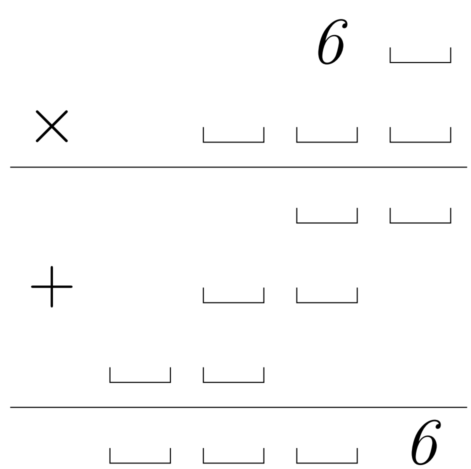
Once we are finished with the third handout, let us switch back to the following problems.

Problem 4 *A herring and a half costs $3. How much do ten herrings cost?*

Problem 5 *Figure out a way to cut the board below into three parts of equal shape and size. You can only cut along the lines.*



Problem 6 *Recover the missing digits.*



Problem 7

- *There are fourteen marbles of three different colors, white, black, and yellow, in a box. The number of the yellow marbles is seven times greater than the number of the white marbles. How many marbles of each color are there in the box?*

- *You pull three marbles out of the box at random. What is the chance that they have the same color?*

Problem 8 *Prove Theorem 1.*