

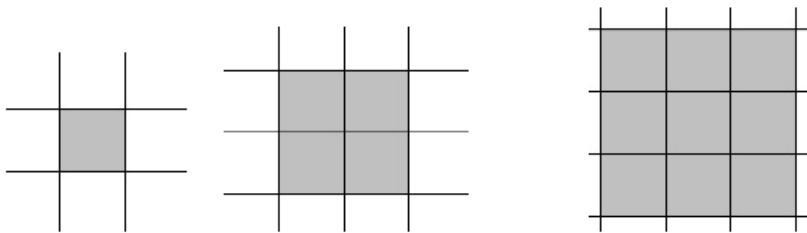
Similar Shapes and Gnomons

May 12, 2013

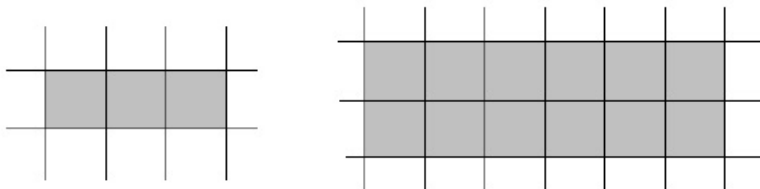
1. Similar Shapes

For now, we will say two shapes are *similar* if one shape is a “magnified” version of another.

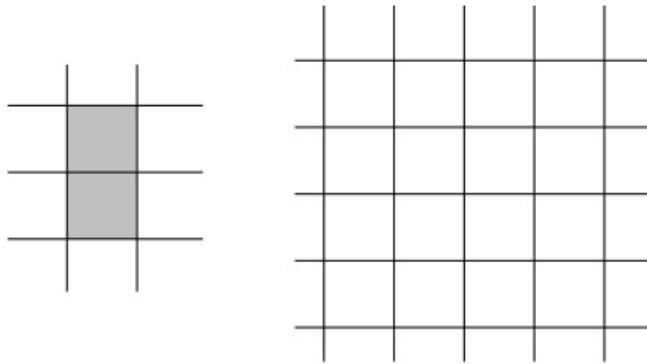
1. In the picture below, the square on the left is of size 1×1 . The square in the middle is *similar* to the first square. The square on the right is also *similar* to the first square with a magnification factor of 3. All three squares are *similar*.



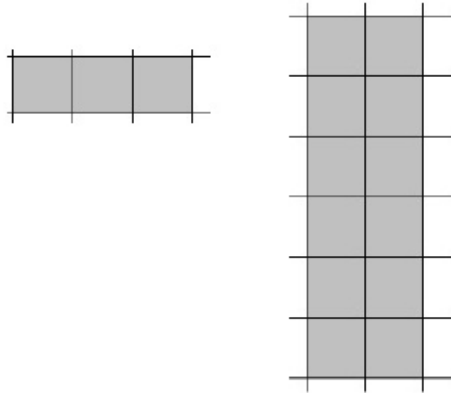
2. Below, the rectangle on the left is of size 3×1 . The rectangle to the right is *similar* to the one on the left: both the width and the length of the rectangle are twice those of the original rectangle on the left.



3. Below is a picture of a rectangle of size 1×2 . Draw and shade in a rectangle on the right that is *similar* to the original rectangle (magnified twice).

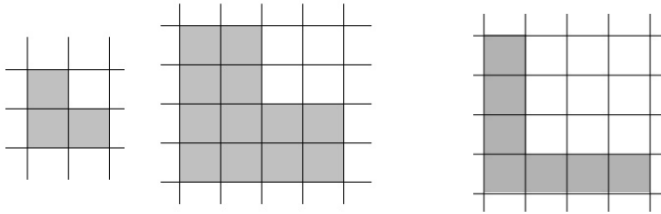


- (a) Do you think that the two rectangles below are similar? Why or why not?



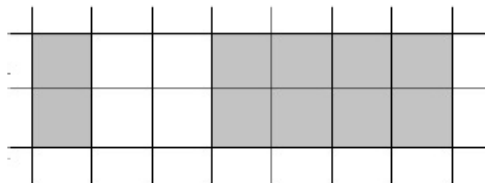
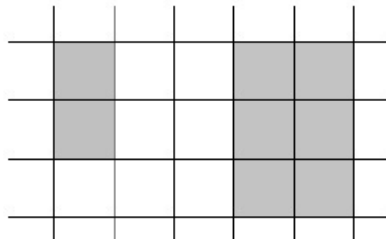
- (b) How are they different compared to the pair above?

4. We can also find *similar* shapes that are not just squares or rectangles. Consider the two L-shapes below:

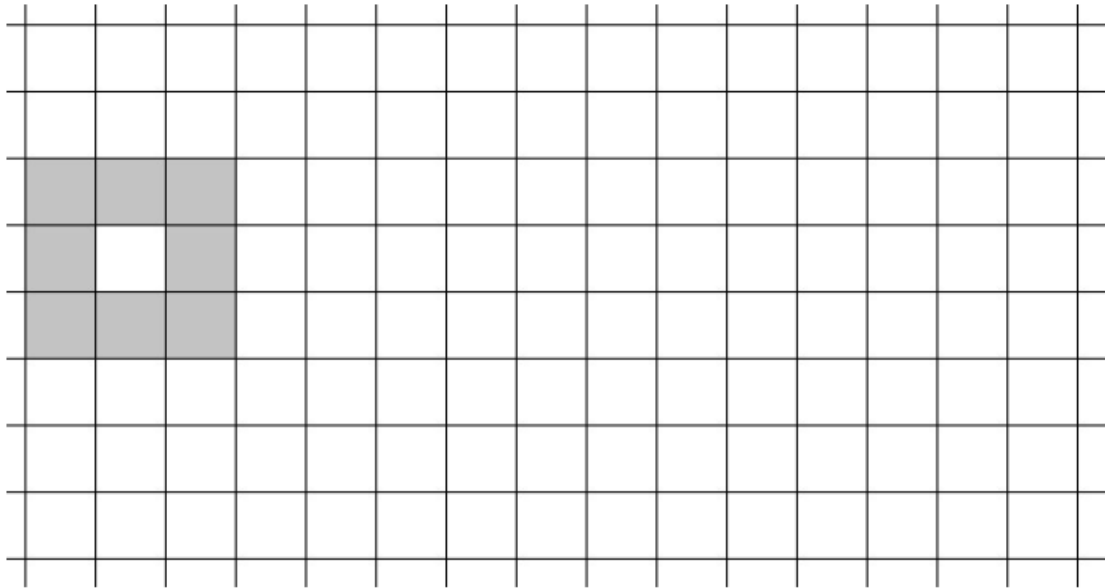
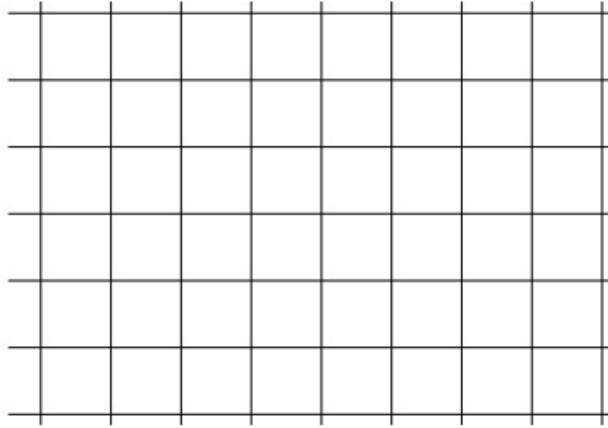
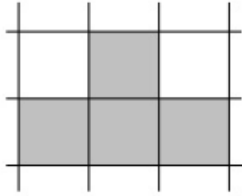


- (a) Can you say that the “L” shapes have the same shape?
- (b) What about the size?
- (c) Do you think they are similar? Why or why not?
5. When trying to decide if two shapes are *similar* by our definition, think about the following questions:
- (a) Does the larger shape look like a magnified version of the smaller?
- (b) If the width of the shape is magnified by a certain factor, is the height also magnified by that same factor?

Are either of the following pairs of rectangles similar?



6. Use the grid to the right of the shapes to draw a *similar* shape that looks like the original, magnified twice.

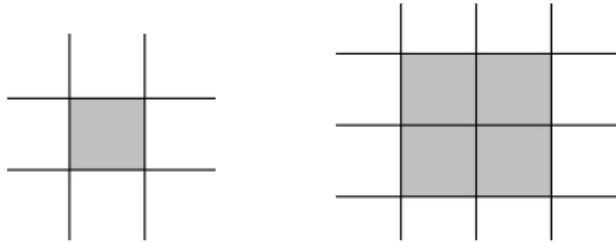


2. Gnomons

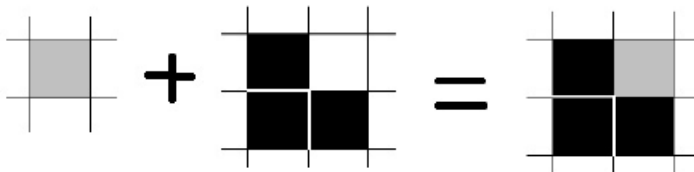
Suppose we have a shape we want to magnify. What is a “gnomon?”

A gnomon is a piece that you can attach to the original shape to create a shape that is *similar* to the original.

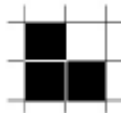
1. Consider two similar squares from the first page:



What piece could we attach to the square on the left to make the square on the right?
Below is one possibility:

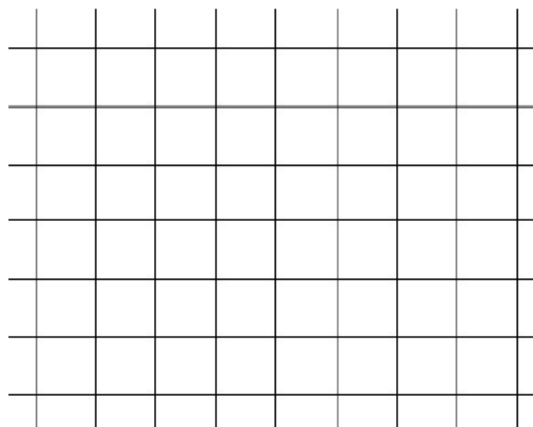


When the dark-colored L-shape is attached to the original 1×1 square, the new square is twice as big.

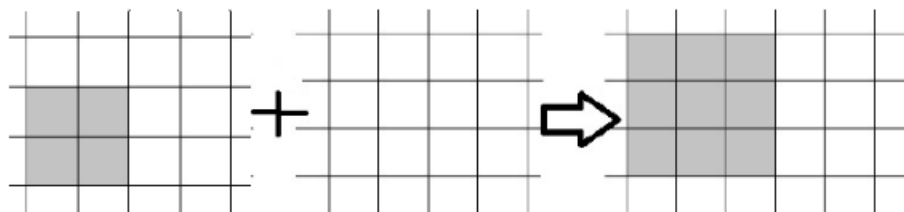
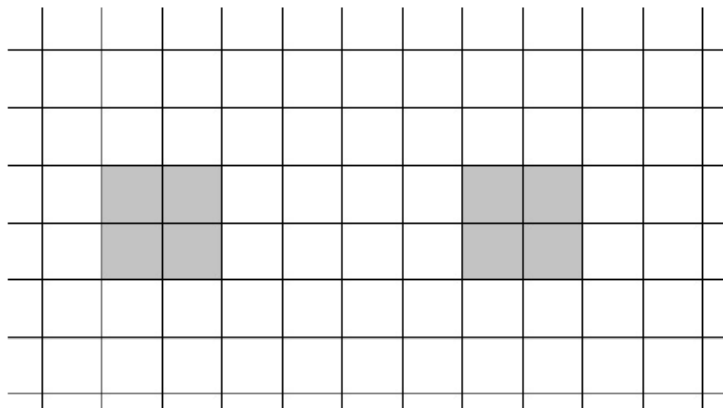


That L-shape is therefore known as a *gnomon* for the square.

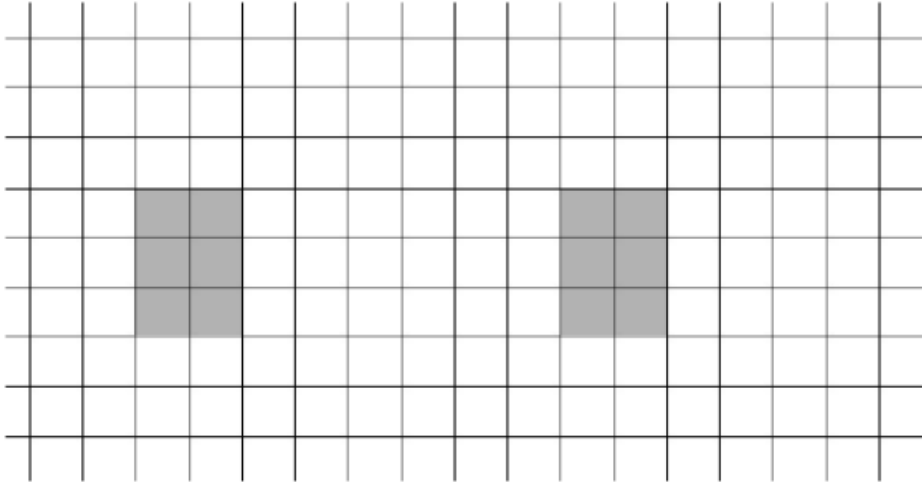
(a) Find another *gnomon* that makes a 1×1 square into a 2×2 square:



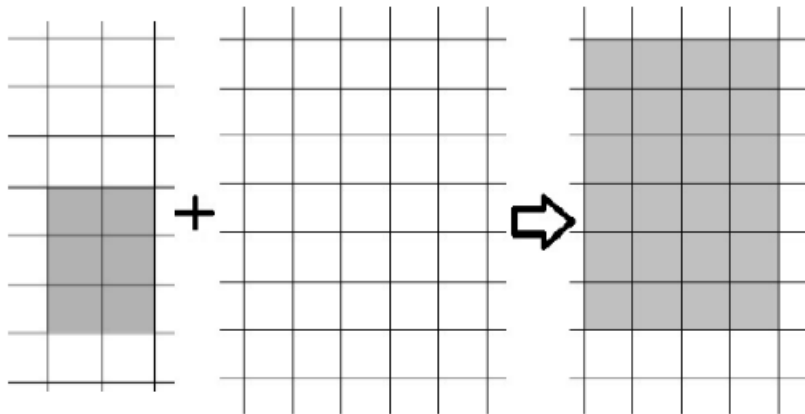
(b) We learned above that a 2×2 square and 3×3 square are also similar. Find the piece (gnomon) that can be added to the 2×2 square to make it a 3×3 square:



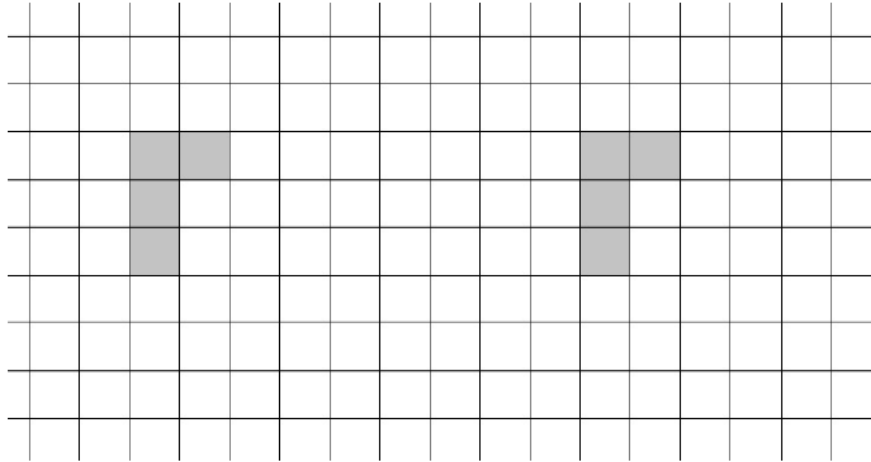
2. Find a gnomon for this rectangle that magnifies it by a factor of two (the resulting *similar* rectangle should be twice as big as the original). You can use the grids below.



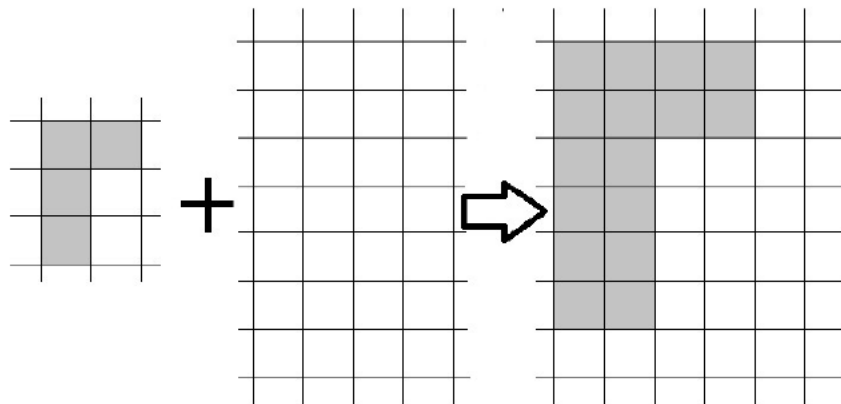
- (a) So what does the gnomon look like? Draw it in the blank spot below:



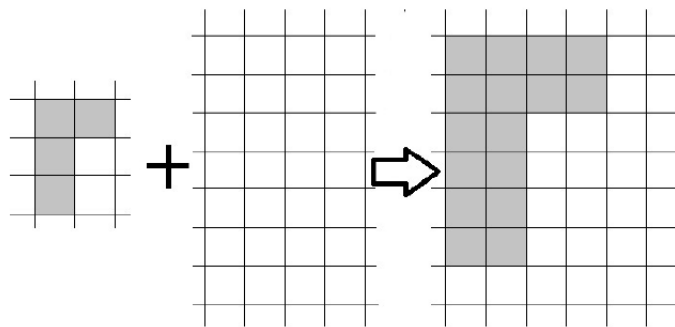
3. Find a gnomon that, when attached to the shape below, creates a *similar* shape that is the original magnified by a factor of 2.



- (a) What does the gnomon look like? Fill in the blank spot below

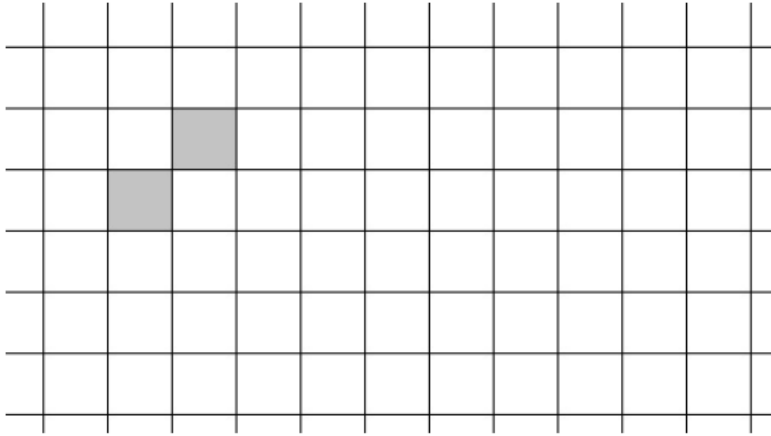


- (b) Draw a different gnomon. (Hint: start by tracing the original in the new shape.)

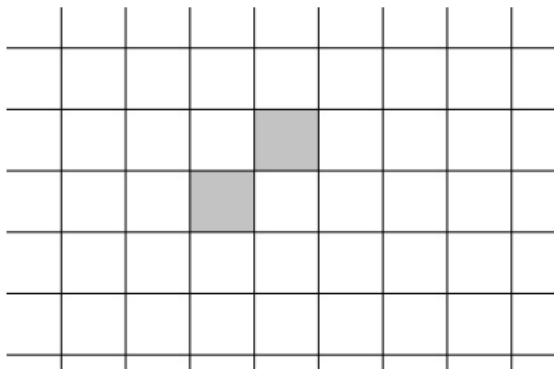
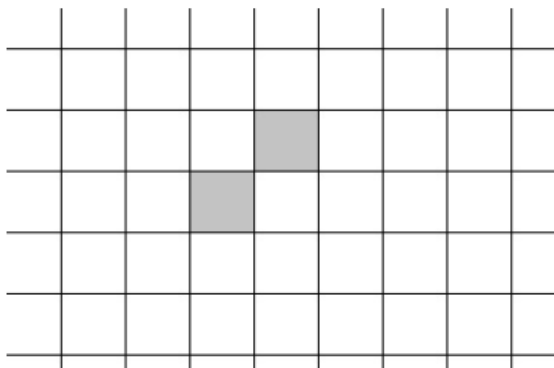


3. Disconnected Gnomons

1. Next to the shape below, draw a similar shape that is the original magnified by a factor of two:

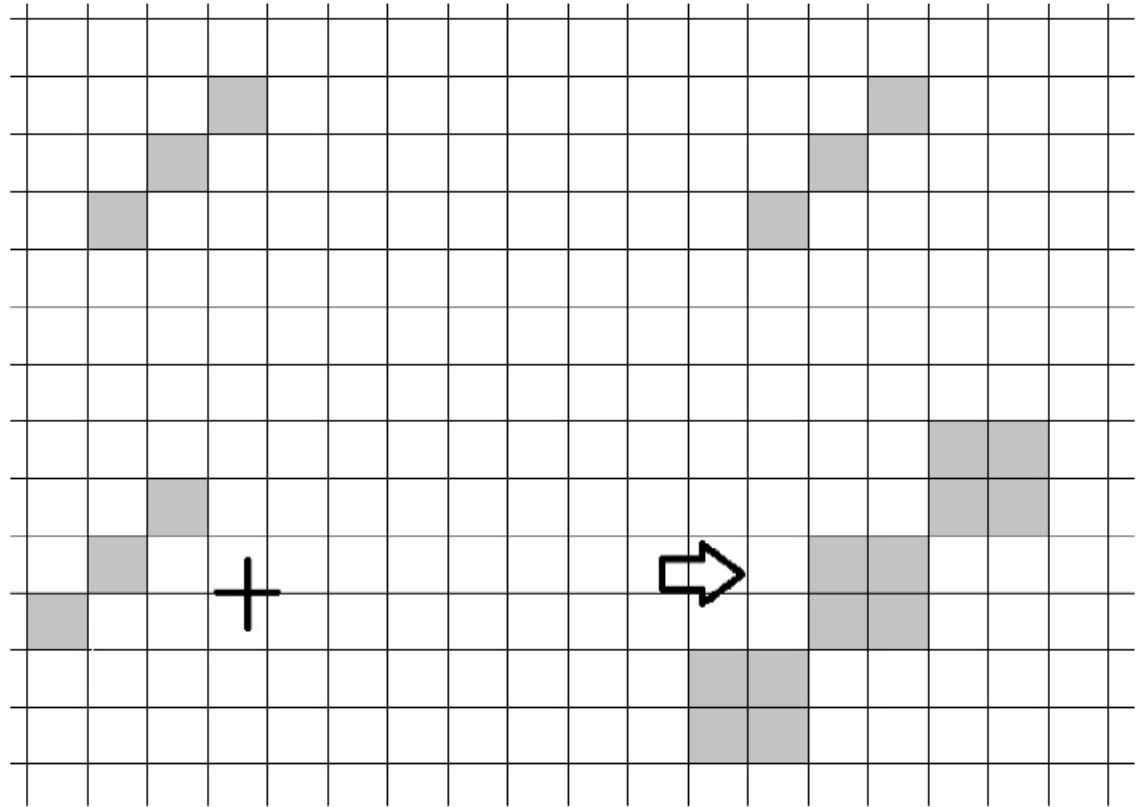


- (a) Looking at the two shapes above, can you find one connected piece that will produce the bigger shape when attached to the original?
- (b) What if you are allowed to add two pieces? Draw them in below. (The two added pieces may be different)

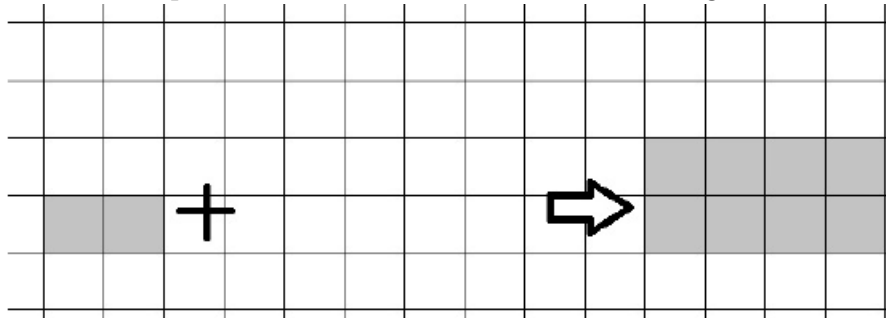


2. As it turns out, some shapes do not have a connected gnomon (a gnomon which consists of a single piece). In these cases, though, sometimes, we can still make shapes *similar* to the original by adding several pieces not connected to each other. We will call the collection of pieces we add to make a *similar* shape a *disconnected gnomon*.

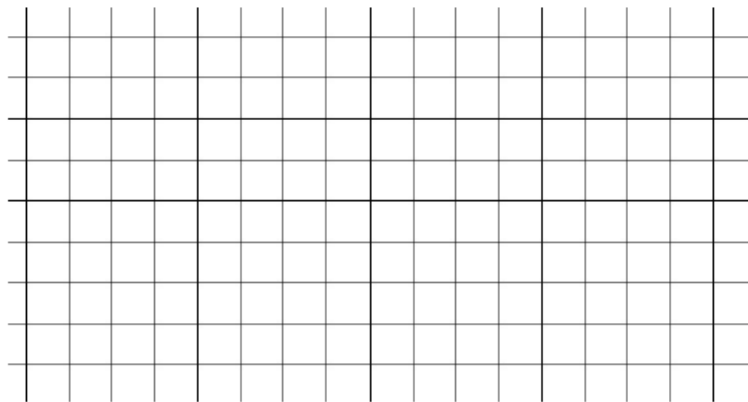
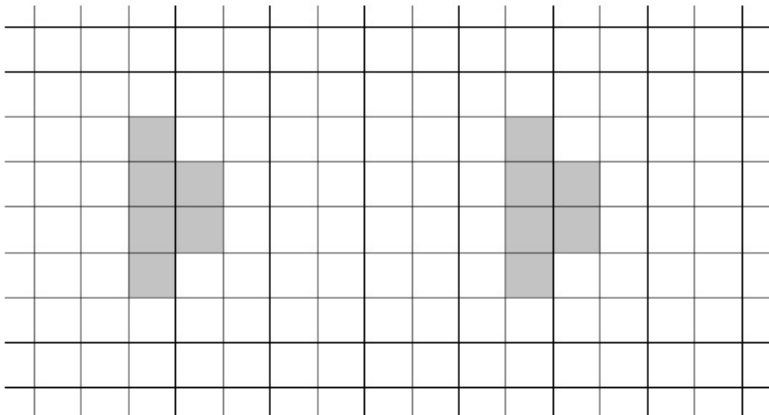
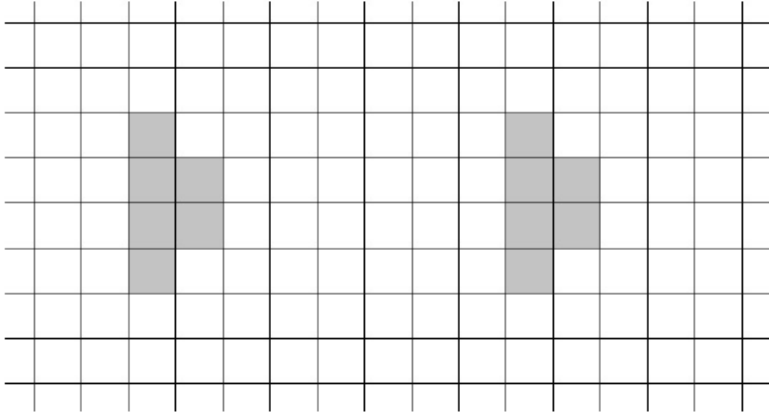
(a) Does this shape have a connected or a disconnected gnomon?



(b) Does this shape have a connected or a disconnected gnomon?

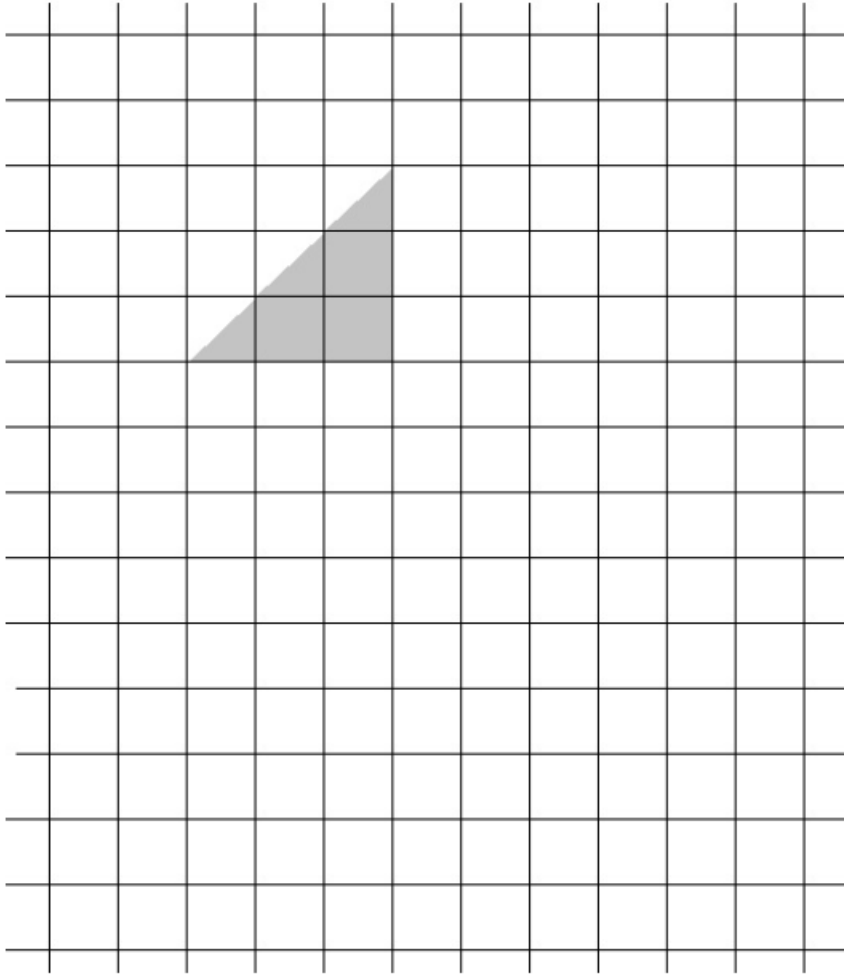


3. Sometimes, a shape can have both a connected and a disconnected gnomon. For the shape below, create a similar image on the right with a magnification factor of 2. Can you find one connected gnomon and one disconnected gnomon for the shape below?



4. Challenge

1. Find two gnomons for the following triangle, which has two equal sides. (If it seems confusing, remember that this triangle is exactly one half of a square cut diagonally.)



5. Homework

Come next week with a shape (not a square or rectangle) with a connected gnomon that, when connected to the original shape, *triples* the size of the shape. Draw the original shape, gnomon, and the larger *similar* shape separately. It may help to use graph paper.