

Topics in Plane Geometry

May 2, 2013

Perpendicular Lines

Right angle = 90°

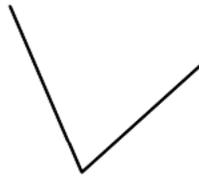
Acute angle $< 90^\circ$

Obtuse angle $> 90^\circ$

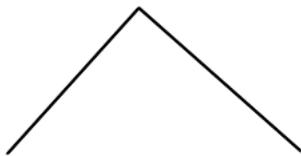
1. Identify the angles below and circle the correct label:



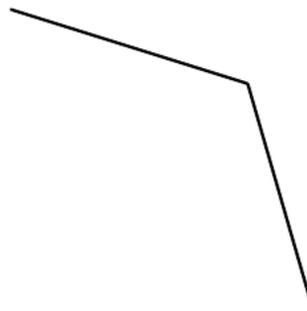
Acute Right Obtuse



Acute Right Obtuse

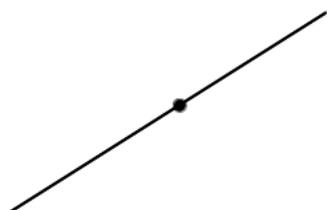


Acute Right Obtuse

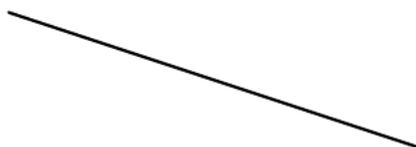
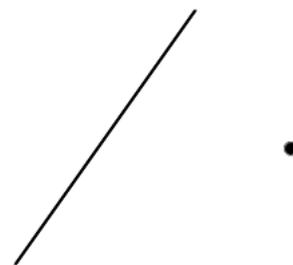
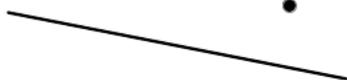


Acute Right Obtuse

2. Draw a line perpendicular to the given line, going through the given point.



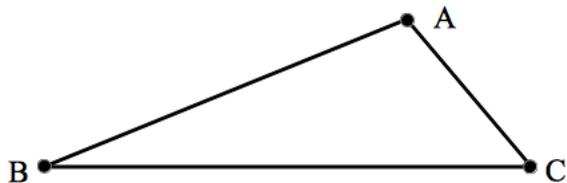
3. Draw a line perpendicular to the given line, going through the given point.



Area of a triangle

1. Area of a Triangle via Area of a Rectangle:

- (a) Draw an altitude AH from the vertex A to the opposite side, BC . Label the point H . (Recall that an *altitude* is the shortest line from a vertex to the opposite side of a triangle)



$\triangle ABC$ is now divided into two right angle triangles, $\triangle AHB$ and $\triangle AHC$.

- (b) Color $\triangle AHB$ and $\triangle AHC$ with *dark* red and *dark* blue.
- (c) On the picture above, draw the line through A which is parallel to BC . Draw the lines through B and C which are perpendicular to this line. Denote points of intersection by D and E .
- (d) What shape is the quadrilateral $BCED$?
- (e) $BCED$ is broken into 4 triangles:
- Shade the triangle equal to $\triangle AHB$ with *light* red.
 - Shade the triangle equal to $\triangle AHC$ with *light* blue.

(f) What portion of the area of the rectangle is taken by $\triangle ABC$? Why?

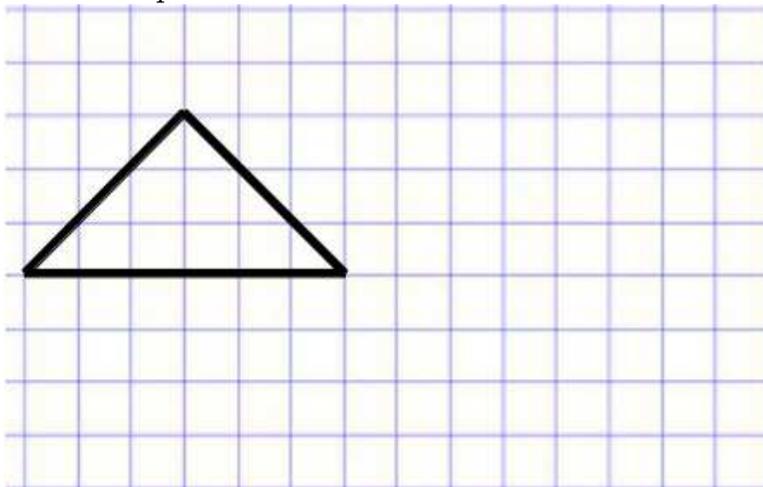
(g) Write down an expression for the area of the rectangle if $AH = h$ and $BC = b$:

$$\text{Area}(\text{rectangle}) =$$

(h) Write down an expression for the area of $\triangle ABC$:

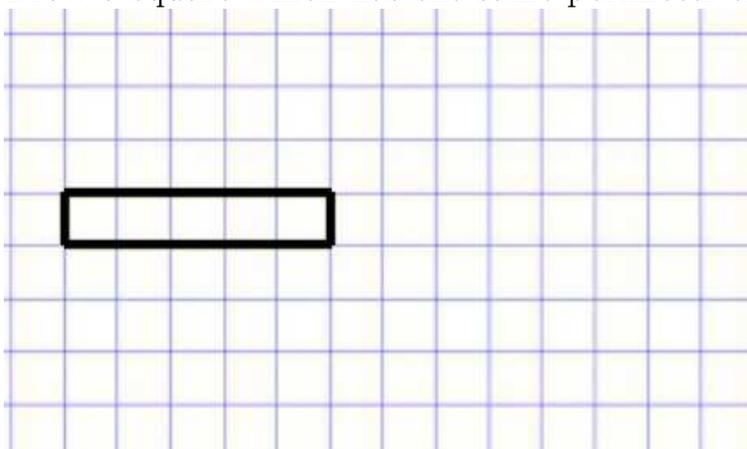
$$\text{Area}(\triangle ABC) =$$

2. Draw a square which has the same area as the triangle below.

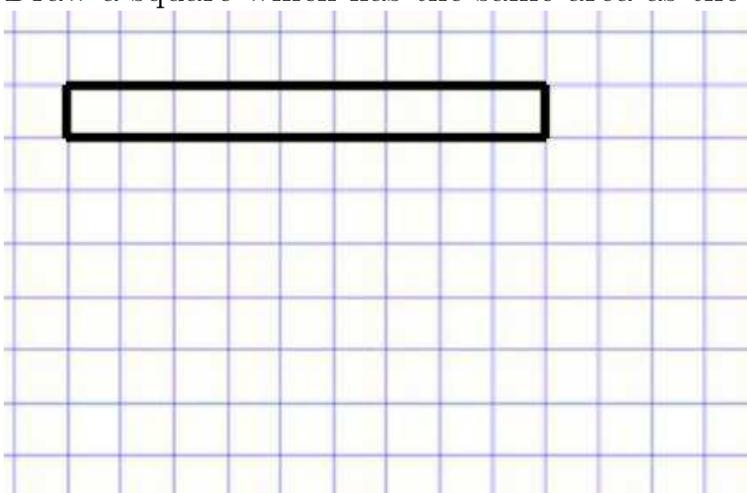


(a) Explain why the areas are the same:

3. Draw a square which has the same perimeter as the rectangle below:



- (a) Explain why the perimeters are the same:
- (b) Which shape has a bigger area: the square or the rectangle?
4. Draw a square which has the same area as the rectangle below:



- (a) Explain why the areas are the same.
- (b) Which shape has a bigger perimeter: the square or the rectangle?

Isoperimetric problem

Suppose you are given 40 meters of fencing and you want to surround a rectangular plot of land whose area is as large as possible. What should the sides of the rectangle be?

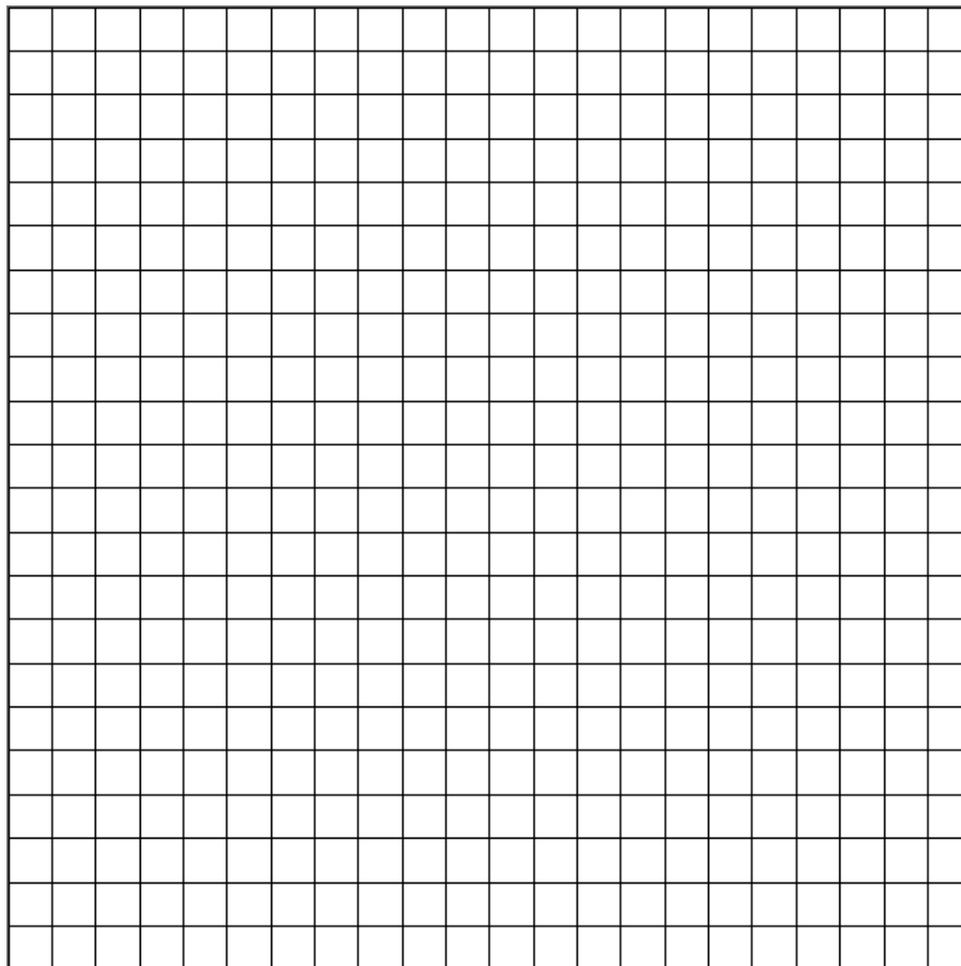
Let's explore how large of a plot we can surround with 40 meters of fencing.

1. If the total length of fencing you have is 40 meters, what is the
2. Start by filling out the table below:

base	height	area
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		

3. What is the shape of the rectangle with the biggest area?
4. Do you think this will hold in general?

5. Use the grid paper below to draw a square with side length 10 and a rectangle with base of length 7 and height 3 so that
- the sides of both square and rectangle are vertical and horizontal;
 - the left lower corner of both the square and the rectangle coincide;



- Which shape has the bigger perimeter?
- Shade the common part of the square and rectangle with red;
- Shade the remaining part of the square with blue;
- Shade the remaining part of the square with yellow;
- Use the picture to explain why the area of the square is bigger than the area of the rectangle;