PROBABILITY II

BEGINNER CIRCLE 4/21/2013

1. WARM-UP: GOING SHOPPING

The math circle instructors go to the store to shop. The set of all items in the store will be called S, and there is a function that assigns a price to each item in the store, P. For instance

$$P(\text{Bazooka Bubblegum}) = .25$$

The price function can also assign prices to shopping carts that are full of items. For instance, Derek's shopping cart (which we will call S_{Derek}) contains Peanuts ($ 1.25), an episode of Seinfeld on DVD ($ 15), and Jonathan ($ 242.43). Therefore,

$$P(S_{\text{Derek}}) = 257.68$$

Problem 1. Morgan, Jeff and Isaac go shopping. There shopping carts contain

$$\begin{aligned} S_{\text{Morgan}} =& \{\text{Waterballoon, Waterbed, Watermelon, Watergun}\} \\ S_{\text{Jeff}} =& \{\text{Waterballoon, Watermelon, Pineapple}\} \\ S_{\text{Isaac}} =& \{\text{Waterbed, Watergun}\} \end{aligned}$$

Upon check out, we discover that

$$\begin{aligned} P(S_{\text{Morgan}}) =& 10 \\ P(S_{\text{Jeff}}) =& 8 \\ P(S_{\text{Isaac}}) =& 7 \end{aligned}$$

What is $P(\{\text{ Pineapple}\})$?

Problem 2. Derek and Jonathan go to the store. Jonathan is planning on going to the beach, so he buys

$$S_{\text{Jonathan}} = \{\text{ Swim Shorts, Surf Board, Snorkle}\}$$

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Derek is a little more safety minded than Jonathan, so he buys everything that Jonathan does, plus a little extra.

$$S_{\text{Derek}} = \{S_{\text{Jonathan}},\ \text{Helmet, Life Jacket, Flashlight}\}$$

Suppose we additionally know on checkout that Helmets, Life Jackets and Flashlights cost a total of \$10, and $P(S_{\text{Jonathan}}) = 25$. What is $P(S_{\text{Derek}})$?

Problem 3. At Office Depot, Jeff and Rachel go shopping for school supplies. Because they have very different styles of mathematics, they buy very different supplies:

$$S_{\text{Jeff}} = \{\text{Whiteboard Marker, Pen, Stapler, Paper}\}$$
$$S_{\text{Rachel}} = \{\text{Chalk, Pencil, Scotch Tape, Paper}\}$$

Morgan can't decide which basket is better, so he buys all the items that are in Jeff's basket or in Rachel's basket. Suppose $P(S_{\text{Jeff}}) = 10$, and $P(S_{\text{Rachel}}) = 7$, and the price of paper is 3. What is $P(S_{\text{Morgan}})$?

Problem 4. Jonathan and Isaac go shopping for dinner. Jonathan buys bananas (\$2), strawberries and cantaloupe and honeydew. Isaac buys strawberries, honeydew, boysenberry (\$ 4) and breadfruit (\$5). Derek is more picky than Jonathan and Isaac, so he buys the fruits that are in both baskets. We know that $P(S_{\text{Derek}}) = 4$ and $P(S_{\text{Jonathan}} = 13)$.

(i) What is $P(\text{Cantelope})$?

(ii) What is $P(S_{\text{Isaac}})$?

(iii) Suppose that Rachel now buys all the foods that are in Jonathan or Isaac's Basket. What is $P(S_{\text{Rachel}})$?

Problem 5. Suppose that Jeff and Derek go shopping, and Isaac buys everything that Jeff or Derek buy, and Morgan buys only things that both Jeff and Derek buy. Explain in a few sentences why

$$P(S_{\text{Isaac}}) = P(S_{\text{Derek}}) + P(S_{\text{Jeff}}) - P(S_{\text{Morgan}})$$

Problem 6. Suppose that Derek and Rachel and Morgan go shopping.

- Jeff only buys things in both Rachel's and Morgan's basket
- Isaac buys everything in either Jeff or Derek's Basket
- Jonathan buys everything in either Derek's or Rachel's basket
- Devon buys everything in either Derek's of Morgan's Basket
- Clyde buys everything in either Jonathan's or Devon's Basket.

Which two people necessarily bought the same items? Explain your solution in a couple of sentences or pictures.

2. SETS!

A set is a group of things, none of which are repeated. In this section, we look at some relationships that sets can have with each other.

Definition 1. The **union** of two sets, $A \cup B$, is the set of all things that are in A or in B

Definition 2. The **intersection** of two sets, $A \cap B$, is the set of all things in A and in B

Definition 3. The size of a set A is the number of elements in it, and is written $|A|$.

Definition 4. The empty set $\emptyset$ is the set with no elements in it

Problem 7. Let $A = \{1, 3, 5, 4, 6\}$ and $B = \{1, 2, 3, 7, 8, \}$

(i) What is $A \cap B$

(ii) What is $A \cup B$

Problem 8. Let A be the set of all animals with fur, and B be the set of all animals that fly. Name some animals in $A \cap B$.

Problem 9. Suppose that none of the elements in A are in B.

(i) What is $A \cap B$

(ii) What is $|A \cup B|$ in terms of $|A|$ and $|B|$

Problem 10. Let S denote the set of all rolls made with two dice. Let $S_{[2]}$ be the subset of all rolls that are a 2, and $S_{[4]}$ be the set of all rolls that have faces totaling to 4. For this question, when you want to say the roll where the first die shows a 2, and the second die shows a 5, write (2,5)

(i) What is the set of all dice that have faces totaling to 2 or 4?

(ii) What is the size of that set?

Problem 11. Again, let S denote the set of all rolls made with two dice. Let S_{E1} be the set of all rolls where the first die has an even value, and S_{E2} be the set of all rolls where the second die roll has an even value.

(i) What is the set of all dice rolls where the first die **and** the second die have an even value.

(ii) What is the size of that set?

(iii) What is the set of all dice rolls where the first die **or** the second die have an even value?

(iv) What is the size of that set?

(v) What is the size of the set where neither the first or second die have an even value?

Problem 12. Last week, we looked at the probability function, P, which assigns probabilities to subsets of the possible outcomes, S. The probability function follows the rules that $P(S) = 1$ and $P(A \text{ or } B) = P(A) + P(B)$. Can you rephrase $P(A \text{ or } B)$ in terms of set operations? Explain in sentences.

Problem 13. How would describe the an event being in both A and B in terms of set operations? Explain in full sentences.

Problem 14. Suppose you are rolling 2 die, then

(i) $P($The first die rolling an even number $)$

(ii) $P($The second die rolling an even number $)$

(iii) $P($ The first and second die rolling and even number $)$

(iv) $P($ The first or the second die rolling an even number $)$

Problem 15. Cards are played with a deck of 52 cards. There are four suits of cards (Spades, Hearts, Diamonds, Clubs), each with 13 cards: 1Ace -10, Jack, Queen and King.

(1) How many cards are in the set of cards that are red?

(2) How many cards are in the set are face cards?

(3) How many cards are face cards, and red cards?

(4) How many cards are face cards or red cards?

Problem 16. Justify why $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ in full sentences. What does this mean in terms of probability?

Problem 17. Justify why $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. What does this mean in terms of probability?

Problem 18. Review problem 6 from the warm up and solve it using sets.