

PROBABILITY III

BEGINNER CIRCLE 4/28/2013

1. FUNNY DICE

Three of the math circle instructors decide to have a little game. They select 1 of three funny dice (to be described below) and roll them. Whoever gets the highest number on their die wins the game.

Jeff's die has sides labeled $\{2, 2, 4, 4, 9, 9\}$ Isaac's die has sides labeled $\{1, 1, 6, 6, 8, 8\}$
Derek's die has sides labeled $\{3, 3, 5, 5, 7, 7\}$

Problem 1. What is the value of the average role that Jeff makes?

Problem 2. What is the value of the average role that Isaac makes?

Problem 3. What is the value of the average role that Derek makes?

Problem 4. If Derek and Jeff both roll, what are the odds that Derek beats Jeff?

Problem 5. If Isaac and Jeff both roll, what are the odds that Jeff beats Isaac?

Problem 6. If Derek and Isaac both roll, what are the odds that Isaac beats Derek?

Problem 7. If all three roll with the dice, who is most likely to win?

2. PROBABILITY AND AND

When we have two independent probability events occurring, the probability that both events occur is equal to the product of their probabilities. For example, if I have a red die and a green die, the probability that the red die rolls a 1 and the green die rolls a 5 is

$$P(\{\text{Red Rolls a 1}\}) \times P(\{\text{Green Rolls a 5}\}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Note that this is very different from asking “If I have two die, what is the probability that I roll a 1 and a 5” which $\frac{2}{36}$.

Problem 8. A deck has 56 cards. The cards come in 4 suits, (Diamonds, Clubs, Hearts and Spades), of which Diamonds and Hearts are red, and Clubs and Spades are Black. Finally, cards take on the values 1 – 10, J, Q, K . What is the probability of drawing a red card?

Problem 9. Suppose that you have two different decks of cards. What is the probability of drawing a red card from the first deck, and a black card from the second deck?

Problem 10. Suppose that you have just one deck of cards. What is the probability of drawing a red card, and then drawing a black card from the same deck?

Problem 11. Suppose you have two decks of cards, and you draw one card from each deck. What are the odds that you have drawn a red card and a black card?

Problem 12. Suppose you have one deck of cards, and you draw two cards. What are the odds that you have drawn a red card and a black card?

Problem 13. Suppose you roll two dice. What are the odds that they have the same value?

Problem 14. Suppose that every time you give Jonathan a quarter, there is a $\frac{1}{4}$ chance that he does a jig. If you give Jonathan a quarter, what are the odds that he does not do a jig?

Problem 15. Suppose you roll 3 dice. What are the odds that you get a pair –that means that at least two of the dice have the same value? (Hint! It may be easier to calculate the probability that you do not get a pair.)

Problem 16. Suppose you roll 4 dice. What are the odds that you roll a pair?

Problem 17. Suppose you draw two cards from the deck. What are the chances that they have the same value (1 – 10, J , Q , K)?

Problem 18. Suppose you draw 4 cards from the deck. What are the chances that at least 2 of them have the same value?

Problem 19. How many cards must you draw from the deck before you are guaranteed that at least 2 of them have the same value?

3. MORE FUNNY PROBLEMS

Problem 20. Rachel and Morgan have a really awesome pair of dice. They don't have the same numbers on them as normal dice. However, when they roll the dice, the chances of the sum of digits being n is exactly the same as the chances of rolling an n with a pair of normal dice. So for instance, the probability of rolling a 4 is $\frac{3}{36} = \frac{1}{18}$. However, we know that the largest numbers on Rachel's dice are an 8 and a 6. (If you are not clear what this problem means, ask an assistant)

- (i) Since Rachel's dice has a 8, what is the largest value that can appear on Morgan's dice?

- (ii) How many ways are there to roll an 11 with normal dice? Consequently, how many 3's must there be on Morgan's dice?

- (iii) How many ways are there of rolling a 10 with normal dice? Consequently, how many 2's must there be on Morgan's dice? (don't forget about the 6 and the 4 on the other dice!)

- (iv) What must the remaining number on Morgan's dice be. Write out what Morgan's dice is.

- (v) Write out Rachel's dice.

Problem 21 (Monty Hall). Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? Explain your solution using math.

Problem 22. Isaac offers to play the following devilish game with you: He flips a coin. If you get a head, he will give you a pencil. If you get a tails, he flips the coin again. If you get a head, he gives you two pencils. Otherwise, he flips the coin again. If you get a head, he gives you 4 pencils, otherwise he flips the coin again. He continues doing this, until he gives you pencils. On average, how many pencils does he give you ? Why is it a bad idea to play this game with Isaac?

Problem 23. A 4 of a kind in poker is when you draw 4 cards that are the same value out of the deck of 52. Suppose you draw 5 cards from the deck. What is the chance that you get a 4 of a kind? (You don't need to simplify your solution, as long you explain how you got your solution)

Problem 24. How many people do you need in a room before the probability of two of them having the same birthday is over $1/2$? (You may instead find the probability that n people in a room have different birthdays)