

MORE ON DISTANCES

MATH CIRCLE (INTERMEDIATE) 04/28/2013

In mathematics, a *distance* is a certain function which tells you how far two objects are in a certain sense. You formulated a more precise definition of a distance last week.

- (1) Define a distance between two line segments of unit length on the plane. Show that your definition obeys the four properties of distance.
(If your definition fails to meet one or more of the necessary properties, you'll have to try a different definition!)

- (2) Suppose Jeff wants to define the “distance” between two segments on the plane as the smallest of the four distances between their four endpoints. Show that this is not a good definition.

(3) Remember the idea of a *taxicab distance*? Let's work with that a bit.

(a) Find the taxicab distance between the following points:

(i) $(1,0)$ and $(1,7)$

(ii) $(4,3)$ and $(12,1)$

(b) Can you find a general formula for the taxicab distance between two points $A = (a, b)$ and $B = (c, d)$?

(4) **Unit circles.**

(a) Define the following terms:

(i) circle

(ii) unit circle

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- (b) Draw the unit circle on the plane for as many distance functions as you can: “regular” distance, taxicab distance, etc. (See the previous week’s handout.) (There may be a prize for the person who provides the most correct answers...)

(5) Draw the points of intersection of the following circles, and if you can find a numerical value, do so.

(a) A taxicab circle of radius 2 and centered at the origin, and a taxicab circle of radius 4 and centered at $(4, 0)$.

(b) A taxicab circle of radius 2 and centered at the origin, and a “regular” circle of radius 4 and centered at $(4, 0)$.

(6) What is the value of π in taxicab geometry?