Lesson 1: Introduction to Physics and Kinematics

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1 Introduction

Before you start trying to solve any physics problems, it is helpful to understand how one should think about these problems in the first place. What key variables, rules, and ideas are needed to predict how physical systems change?

Science relies on measuring quantities in our universe. It is important to know what you are measuring, how it is defined, and which units you are using.

1.1 Units

We measure physical quantities using units that can be compared with agreed standards. We will use the International System of Units (SI), including meters, seconds, and kilograms. Other systems use units such as inches and pounds.

Unit prefixes help us describe very large or very small quantities. The following table lists selected SI prefixes.

Factor	**Prefix**	**Symbol**
10^{15}	peta-	P
10^{12}	tera-	T
10^{9}	giga-	G
10^{6}	mega-	M
10^{3}	kilo-	k
10^{2}	hecto-	h
10^{1}	deka-	da
10^{-1}	deci-	d
10^{-2}	centi-	c
10^{-3}	milli-	m
10^{-6}	micro-	μ
10^{-9}	nano-	n
10^{-12}	pico-	p
10^{-15}	femto-	f

Notation

$\Delta x = x_f - x_0$ denotes a finite change in a quantity x. An arrow, as in $\vec{v}$, denotes a vector. The symbols $\hat{\imath}$ and $\hat{\jmath}$ denote unit vectors along the positive x- and y-axes. For example, $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$.

1.2 Mass

Mass is a measure of inertia of a physical object. Inertia is a property of an object to resist an attempt to change the way it moves or rests. For example, it is harder to accelerate a loaded truck from a standstill to moving at the speed of 1 meter per second than doing the same to a tennis ball. There also exists gravitational mass, equal to the inertial mass. It took Einstein to explain the reason why the two are equal.

1.3 Time

The word "time" has many different meanings. Time in the more elementary part of physics is a variable used to describe when an event happens as well as the duration of the happening. For example, "I had lunch from 10:30 to 11:30 AM PST." Even this, more elementary, notion of time is quite non-trivial, see problems 1.1 and 1.2.

A different approach is to think of entropy (measure of randomness of a physical system) as of a time arrow. Any closed physical system progresses towards a more probable state, the rate of such progression being perceived as time by an observer. This explains why time flows one way, but not the other, despite the fact that time running back, instead of forth, does not change correctness of most physical equations. A notable fact here is that life, considered as self-organization of matter, works against entropy, increasing complexity of the system.

Biological time measures aging of a living creature. For example, you may be familiar with the concept of a dog year. Psychological time measures the speed at which a brain receives, processes, and stores incoming information. This one is similar to the clock-speed of a computer.

To summarize, if one digs deep, they find out that we don't know what time is, how many dimensions it has (it may have more than one), or whether it even exists. Never mind that, we will use the most elementary notion as in having lunch from 10:30 to 11:30 AM PST.

Problem 1.1. A definition attributed to Einstein is, "Time is what a clock measures."[1] You are in LA talking to your buddy in NYC. Do correctly set local clocks in LA and NYC show the same time? Explain how different clock readings can describe the same instant.

[1]This wording is quoted in William D. Phillips's lecture, *Time, Einstein, and the Coolest Stuff in the Universe*, available from NIST.

Problem 1.2. When people cross the International Date Line (IDL) moving west, they advance the calendar by one day. When they cross in the opposite direction, they move the calendar back by one day. The IDL runs from the North Pole to the South Pole.

You have just run 366 laps around the North Pole. Do the 366 date-line crossings themselves make you 366 days older or younger? Distinguish changes in calendar dates from the actual time spent running.

1.4 Position and Displacement

Position specifies an object's location relative to an origin in a chosen coordinate system. In two dimensions, a position vector has two coordinates, $\vec{r} = (x, y)$. In three dimensions, it has three coordinates.

Displacement is a change in position: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$. For example, if the coordinates are measured in meters, moving from $\vec{r}_1 = (0, 0)$ to $\vec{r}_2 = (1, 0)$ gives $\Delta\vec{r} = (1, 0)$ and horizontal displacement $\Delta x = 1\,\mathrm{m}$.

Problem 1.3. Could your total displacement be zero while the total distance you traveled is nonzero? Draw such a scenario.

1.5 Velocity

Speed describes how fast an object moves. Velocity is a vector whose magnitude is the speed and whose direction is the direction of motion. For example, moving at 5 m/s in the positive y-direction gives a different velocity from moving at 5 m/s in the negative x-direction.

The average velocity over a time interval Δt is

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}.$$

For motion along a single coordinate axis, we use signed scalar coordinates and velocities. Along the x-axis,

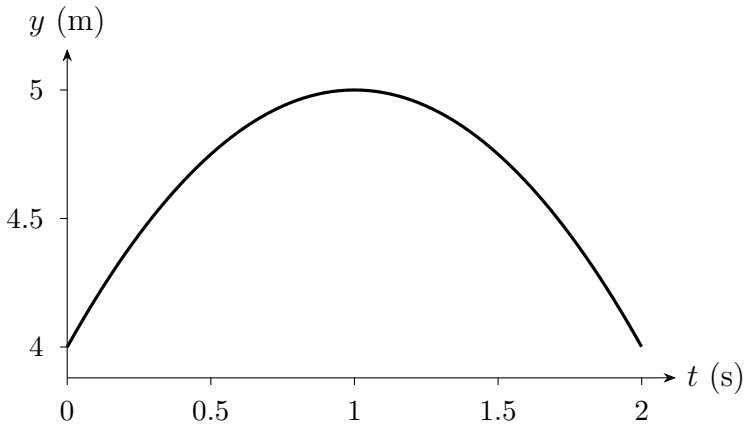
$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}.$$

Thus, average velocity is the average rate of change of position over the time interval.

Problem 1.4. The vertical position of an object is given numerically by

$$y(t) = -(1-t)^2 + 5,$$

where t is measured in seconds and y in meters. Find its average velocity over $[0, 1]$. Draw the line through the two corresponding points on the graph. This line has slope equal to the average rate of change over that interval.

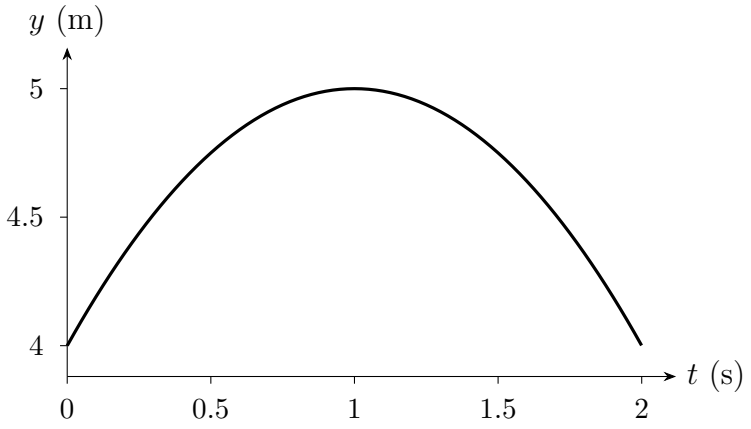


The line you drew is a **secant line**: a line through two distinct points of a curve. A secant can intersect a curve at additional points as well.

Problem 1.5. For the function in Problem 1.4, hold one endpoint at $t = 0.5$ and consider successively smaller intervals:

$$[0.5, 1], \qquad [0.5, 0.8], \qquad [0.5, 0.6], \qquad [0.5, 0.51].$$

Find the average velocity on each interval. Sketch the corresponding secant lines on the graph below. What happens to the slopes as the other endpoint approaches $t = 0.5$?



As the second point approaches the fixed point, the secant line approaches a **tangent line**, provided the limiting slope exists. At a differentiable point, the tangent passes through that point and has slope equal to the instantaneous rate of change. A tangent line can cross the curve at the point of tangency and can intersect it at other points.

For motion along one axis, instantaneous velocity is

$$v(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}.$$

Problem 1.6. Find the average velocity of $y(t)$ from Problem 1.4 over $[0, 2]$. At what time, if any, does the instantaneous velocity equal this average velocity?

Perelman's Tram Problem

Problem 1.7. One day, a great Russian math teacher, Yakov Perelman, was walking down a street in St. Petersburg at a constant speed. He noticed that successive trams traveling in the same direction overtook him every twelve minutes. Trams traveling in the opposite direction, at the same speed, passed him every four minutes.

Assume that trams leave both terminals at the same regular time interval and all travel at the same constant speed without stopping.

(a) Let v be Mr. Perelman's speed, and let w be the speed of a tram. Find the ratio w/v.

The ratio $w/v =$ _____.

Problem 1.7 continues on the next page.

Perelman's Tram Problem (continued)

Problem 1.7(b). What was the time interval between consecutive trams leaving the same terminal?

The time interval was _____ minutes.

1.6 Acceleration

If velocity describes how position changes with respect to time, acceleration describes how velocity changes with respect to time. Average acceleration over a time interval Δt is

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t}.$$

Instantaneous acceleration is

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t}.$$

For motion along a single axis, the corresponding signed scalar formulas are

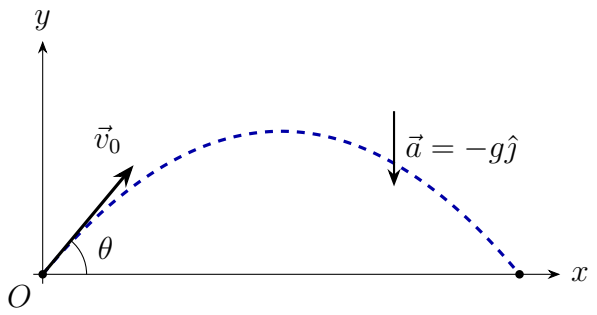
$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}, \qquad a(t) = \lim_{\Delta t \to 0} \frac{v(t + \Delta t) - v(t)}{\Delta t}.$$

Problem 1.8. Can an object's speed remain unchanged while its net acceleration is nonzero? If so, describe such a scenario and draw it.

2 Kinematics

2.1 Projectile Motion

Projectile motion describes the motion of an object after it is thrown or launched and is subject only to gravity. Near Earth's surface, we approximate its acceleration as a constant downward vector of magnitude $g = 9.8\,\text{m/s}^2$.



Assumptions

In Problem 2.1, assume that the given acceleration *vector* is constant. For the projectile motion problems, neglect air resistance, take upward as positive, and use $a_x = 0$ and $a_y = -g$, with $g = 9.8\,\text{m/s}^2$. In uniform circular motion, the acceleration has constant magnitude but changes direction.

Problem 2.1. Given an initial velocity $\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}$ and a constant acceleration $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$ over an elapsed time Δt, find the final velocity $\vec{v}_f$. Write equations for the x- and y-components and then combine them into a vector equation.

Problem 2.2. Newton throws an apple at an angle θ above the horizontal. Model the apple as a particle launched from ground level and landing on the same flat ground. It takes the apple a total time Δt to hit the ground.

Newton measures the maximum height H above the ground and the horizontal range R. How can he use H, R, and Δt to find the initial velocity components and initial speed v_0?

Hint: Think about average velocities and the time spent ascending.

As part of your reasoning, establish the constant-acceleration relation

$$\Delta\vec{r} = \tfrac{1}{2}(\vec{v}_f + \vec{v}_0)\Delta t.$$

Problem 2.3. Newton repeats the experiment the next day, but he forgets his stopwatch. He can measure displacements, and he brings a small ballista with a fixed exit speed v_e, positioned at an angle θ above the horizontal.

Let $\Delta y = y_f - y_0$ be the apple's vertical displacement from launch to impact. Find its horizontal and vertical velocity components immediately before impact, when it is moving downward. Then simplify your answer for equal launch and landing heights.

Hint: Use $a_x = 0$ and $a_y = -g$. Eliminate Δt from the constant-acceleration velocity and displacement equations to derive, along a chosen axis,

$$v_f^2 = v_0^2 + 2a\Delta x.$$

The preceding problems develop the first three constant-acceleration relations below. In every equation, Δt is elapsed time from the initial instant.

Kinematics equations

For a constant acceleration vector:

$$\vec{v}_f = \vec{v}_0 + \vec{a}\,\Delta t,$$
$$\Delta\vec{r} = \tfrac{1}{2}(\vec{v}_f + \vec{v}_0)\Delta t.$$

For signed components along the coordinate axes:

$$v_{fx}^2 = v_{0x}^2 + 2a_x\Delta x, \qquad v_{fy}^2 = v_{0y}^2 + 2a_y\Delta y.$$

For projectile motion with $a_x = 0$ and $a_y = -g$:

$$\Delta y = v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2, \qquad \Delta x = v_{0x}\Delta t.$$

Problem 2.4. Derive the last two equations in the box using the constant-acceleration relations above them.

Problem 2.5. An archer shoots an arrow off a cliff with an initial speed of 30.0 m/s at 15.0° above the horizontal. It lands 25.0 m below the cliff edge. Find (a) the flight time and (b) the magnitude and direction of its velocity just before impact. Neglect air resistance.

Problem 2.6. An archer fires an arrow from the ground so that it passes through two hoops, which are both at a height h above the ground. The arrow passes through the first hoop one second after it is launched and through the second hoop another second later. What is the value of h?

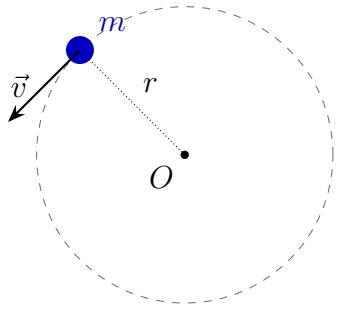
Problem 2.7. A cannon sits atop a ledge of height h above level ground. Given a fixed initial speed v_0, at what launch angle θ above the horizontal should it fire its cannonball to maximize the horizontal range Δx?

Problem 2.8. A person jumps vertically upward from the ground with initial speed v_0. Neglect air resistance. What percentage of the total time in the air does the person spend within the top 4% of the maximum height?

2.2 Uniform Circular Motion

Uniform circular motion is motion at a constant speed along a circular path of fixed radius r.

Problem 2.9. For the example below, draw the direction of the acceleration vector.



In uniform circular motion, acceleration points toward the center of the circle. It is called **centripetal acceleration**. Its magnitude is

$$a_c = \frac{v^2}{r}.$$

Although this magnitude is constant, the acceleration vector changes direction as the object moves around the circle.

Problem 2.10. What is the period T, the time needed for the ball to complete one revolution around a circle of radius r at speed v?

Problem 2.11 (Bonus). Prove that the acceleration vector in uniform circular motion points inward, toward the center of the circle.