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**Problem 1** *There exist two kinds of inhabitants on the planet HD 85512 b<sup>1</sup>, Barabasses and Carabasses. Each Barabas is a friend to seven Barabasses and ten Carabasses. Each Carabas is a friend to nine Barabasses and six Carabasses. Are there more Barabasses or Carabasses on the planet?*

**Problem 2** *There are 2013 points on a circle, 2012 black points and one red point. Consider all the possible polygons with vertices at these points. Which number is larger, the number of polygons without the red vertex or the number of polygons with it?*

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<sup>1</sup>Orbiting the “orange dwarf” star Gliese 370 approximately 36 light-years away from the Earth in the constellation of Vela (the Sail).

**Problem 3** *There are 12 people in a room. Some of them always tell the truth, others always lie. One of them says, “There is no truthful person in this room.” Another says, “There is no more than one truthful person in this room.” Says the next one, “There are no more than two truthful persons in this room”, and so forth. The last person says, “There are no more than eleven truthful persons in this room.” How many truthful persons are there in the room?*

**Problem 4** *Prove that  $x^2 - y^2 = (x - y)(x + y)$ .*

**Problem 5** *All the three sides of a right triangle in the Euclidean plane have integral lengths. Prove that at least one of them is divisible by two and at least one is divisible by three.*

**Problem 6** *There are three switches in a room, each connected to a light bulb in the other room. The switches are marked “on” and “off”. How to figure out what switch is connected to what light, if you are only allowed to enter the second room once? Hint: think “outside of the box.”*

**Problem 7** Find the smallest positive integer such that its last two digits are 56, it is divisible by 56, and the sum of its digits equals 56. Using a calculator at some point may help to solve this problem.

**Problem 8** Does there exist a positive integer such that its last two digits are 11, it is divisible by 11, and the sum of its digits equals 11? Why or why not?