Extensive Form Games

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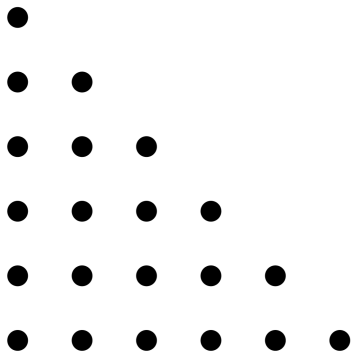
Warm-up 1. There are 6 purple pegs, 5 yellow pegs, 4 red pegs, 3 green pegs, 2 blue pegs, and 1 orange peg to be placed on a triangular peg board. In how many ways can the pegs be placed so that no (horizontal) row or (vertical) column contains two pegs of the same color?



Warm-up 2 (Expected Value Practice).

(a) Let X be a random variable recording the result of one roll of a fair 20-sided dice. What is $\mathbb{E}[X]$, the expected value of X?

(b) Let Y be the sum of 100 independent rolls of fair 20-sided die. That is,

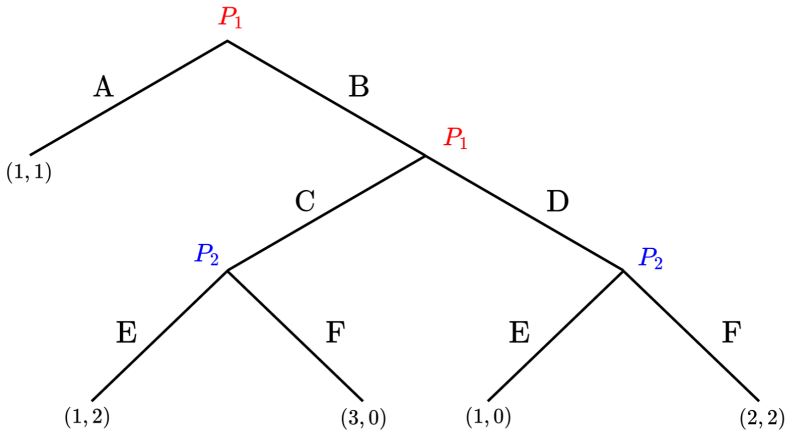
$$Y = X_1 + X_2 + \cdots + X_{100}.$$

Find $\mathbb{E}[Y]$.

(c) You flip a fair coin. If it is tails, you get 0 points and the game ends. If it is heads, you get 1 point, and you get to flip the coin again following the same rules. Let T be the total number of points you collected after the game ends. Find $\mathbb{E}[T]$.

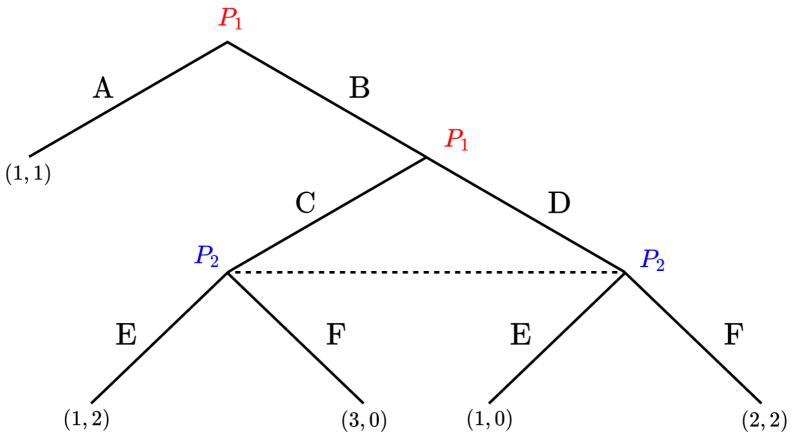
Imperfect Information

Exercise 1. Solve for the SPNE of the following game. Write the strategy pair in the box.



Note that in exercise 1, both players have perfect information. In particular, P_2 knows whether P_1 played A or B, and C or D. But what happens when P_2 no longer can differentiate between P_1 playing C or D?

Exercise 2.



(a) Explain why you can no longer solve for the NE using backwards induction.

(b) How many subgames are there in this game?

(c) Rewrite the smallest subgame you found containing the nontrivial (containing more than one node) information set into strategic form, then solve for its NEs.

(d) Redraw this whole game tree, but replacing the subgame with the expected payoffs of the strategic form game you analyzed in part (c).

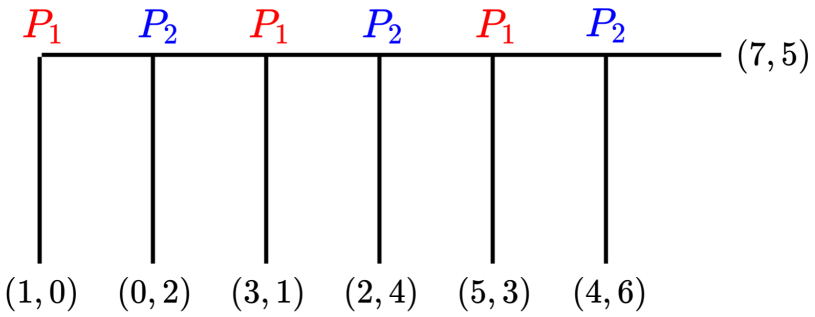
(e) Solve for SPNEs and their payoffs.

(f) Why might the payoffs be lower than in exercise 1? Explain why two rational players cannot just agree to play (BD,F) before the game starts and try for a higher payoff of (2,2).

Theory vs. Practice

In this section, we explore many games that players in real life end up playing differently than they "should" if they were playing optimally. Studies that had participants play these games found that in some cases, human behavior is not always so rational.

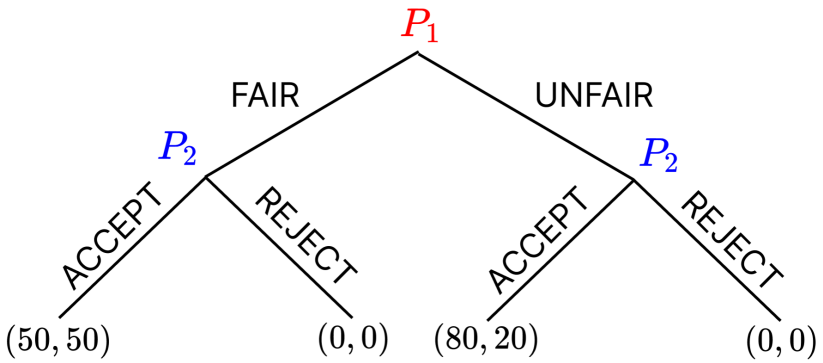
Exercise 3 (Centipede Game).



(a) Solve the game.

(b) Does the SPNE seem convincing to you?

Exercise 4 (Ultimatum). Players in the following game are playing for \$100. P_1 gets to make the decision on how to split the money.



(a) Solve for the SPNE.

(b) What would you do as P_1 if P_2 was your friend? What if they were your enemy?

(c) What would you do as P_1 if you were strangers with P_2?

(d) What would you do as P_2 if you were strangers with P_1 and they gave you an unfair offer?

Trivia

The ultimatum game, created by Werner Güth, Rolf Schmittberger, and Bernd Schwarze, was studied extensively with real players. They found that players belonging to the same social group mostly chose to give fair offers, and offers of less than 30% were often rejected, even though that is an "irrational" play by P_2.

(e) How might players' strategies change if they knew they had to play this game 1000 times repeatedly within the same small circle of players?

Note

There are many more examples of games such as this one, where one's intuition on how players ought to play seem to contradict what the theory says a rational player ought to do. One angle that may explain this discrepancy is that humans evolved to have repeated positive interactions within their communities, lest they be ostracized. Therefore, even when faced with a one-off ultimatum scenario, our brains still intuitively treat the game from the framework of one of many repeated interactions, in which offering an unfair deal would significantly hurt our reputation, which would harm us in the long run.