Powers and Logarithms II **ORMC Intermediate 2**

Oleg Gleizer
prof1140g@math.ucla.edu

More powers

Problem 1 *Put the correct sign, $>$, $<$, or $=$, between the following numbers.*

$$10^{-3} \qquad 2^{-10}$$

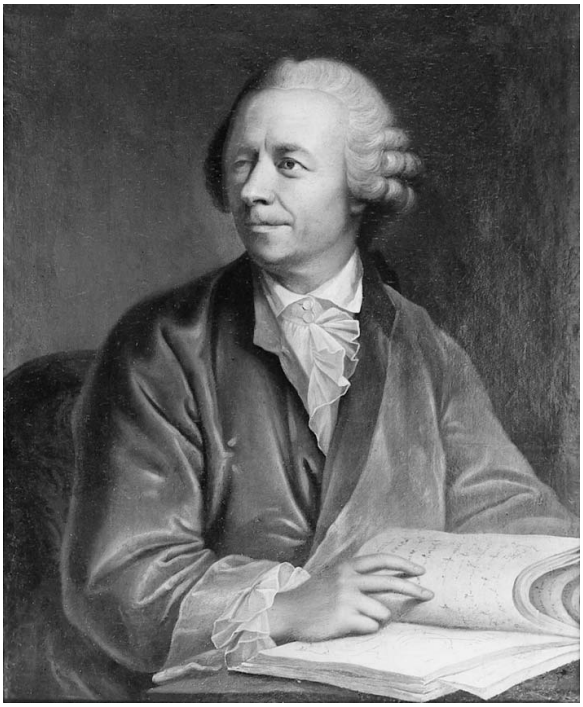
Problem 2 *Given $2^{1000}/2^n = 2^{510}$, find n.*

Problem 3 *Estimate the number of decimal digits needed to write* 2^{100}*.*

Problem 4 *A city population grows 5% every year. How long would it take to triple?*

Logarithms continued

People often need to take logarithms to different bases, but our calculators can compute at most two. The first is the *natural logarithm*, $\ln x = \log_e x$. The number $e = 2.71828\ldots$ is a fundamental constant underlying mathematical description for any type of a chain reaction, from nuclear fusion that powers the stars to an illness spread during an epidemic outbreak. Discovered by one of the greatest mathematicians of all times, Leonhard Euler, the constant is called the *Euler number* (hence the e notation) in his honor.



Leonhard Euler

Problem 5 *Prove the following formula.*

$$\log_b x = \frac{\ln x}{\ln b} \tag{1}$$

Problem 6 *Use formula 1 and your calculator to find the following.*

- $\log_2 100 =$

- $\log_5 21 =$

The other logarithm, less important, but about as widespread as the natural one, is the *common logarithm*, $\lg x = \log_{10} x$. The formula

$$\log_b x = \frac{\lg x}{\lg b} \tag{2}$$

is proven similar to 1.

Problem 7 *Use formula 2 and your calculator to find the following.*

- $\log_3 17 =$

- $\log_5 e =$

Problem 8 *Prove the general formula for a base switch below.*

$$\log_b x = \frac{\log_c x}{\log_c b}$$

Harder problems on logarithms

Problem 9 *Given* $\log_{27} a = b$*, find* $\log_{\sqrt{3}} a^6$*.*

Problem 10 *How is* $\log_b a$ *related to* $\log_a b$*?*

Problem 11 *Given* $\log_7 2 = a$, *find* $\log_{\frac{1}{2}} 28$.

Problem 12 *Given* $\lg 3 = a$ *and* $\lg 11 = b$, *find* $\log_9 297$.