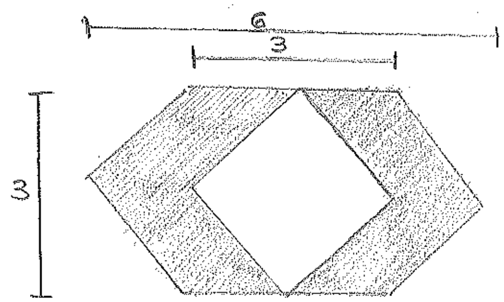
GEOMETRY - FINDING ANGLES II

MATH CIRCLE (INTERMEDIATE) 03/03/2013

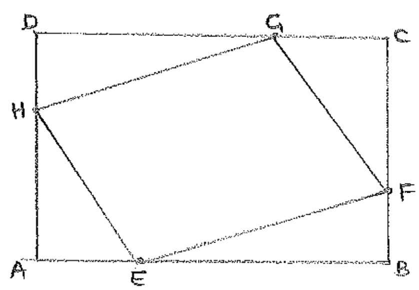
(1) Given an equilateral triangle with side length s, prove that the area of the triangle is $\frac{s^2\sqrt{3}}{4}$.
(Hint: Draw an altitude in the triangle.)

(2) What is the area of an equilateral triangle that has altitude length 12?

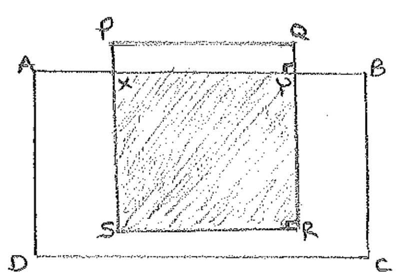
(3) What is the area of the shaded figure?



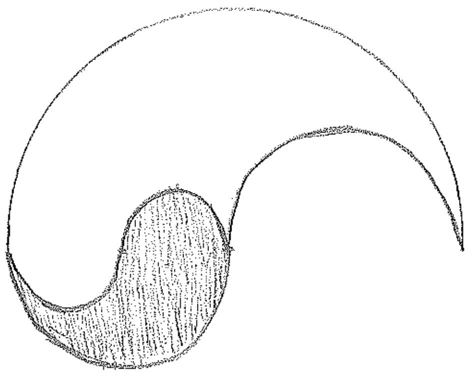
(4) The area of rectangle $ABCD$ is equal to S. Points E, F, G, and H divide the sides of the rectangle in the ratio of 1:2 (see the picture). What is the area of parallelogram $EFGH$?



(5) In the figure, $PQRS$ is a square. The area of the shaded region is equal to half the area of rectangle $ABCD$. What is the length of segment PX?

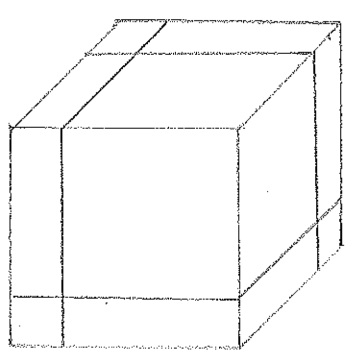


(6) The figure to the right is composed of arcs of semicircles with radii 2, 4, and 8 cm. What fraction of the figure is shaded?

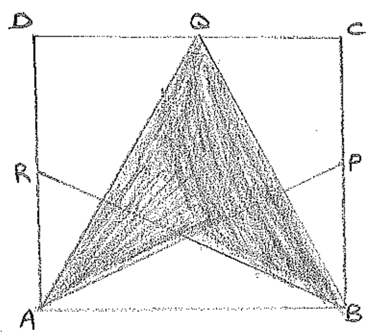


(7) Find the ratio of the area of an equilateral triangle inscribed in a circle to the area of a square circumscribed about that same circle.

(8) A large cube has been cut into eight rectangular prisms as shown in the picture. What is the ratio of the total surface area of the eight pieces to the surface area of the cube?

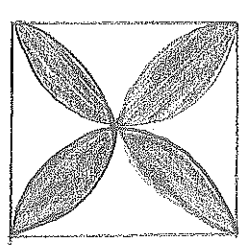


(9) Let $ABCD$ be a square of side length 12. Points P, Q, and R are the midpoints of sides BC, CD, and DA, respectively. What is the area of the shaded region?

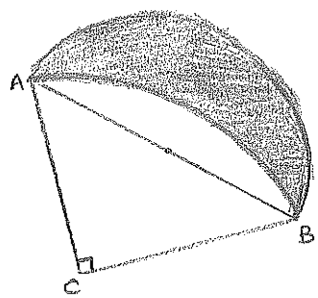


(10) A cow is tied to the outside of a corner of a 20 foot by 15 foot shed with a 30 foot rope. Find the total grazing area. (The cow cannot get inside the shed.)
(Hint: Be careful...what happens on the back sides of the shed? The rope must go around, not through, the shed!)

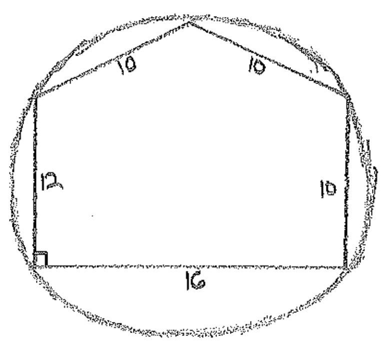
(11) Given the square in the figure with side length 4 and four semicircles which the have the sides of the square as their diameters as shown, find the area of the 'leaves' which are shaded in the diagram.



(12) Find the shaded area, given that $\triangle ABC$ is an isosceles right triangle. The midpoint of AB is the center of semicircle AB, point C is the center of quarter circle AB, and $AB = 2\sqrt{2}$.



(13) Find the area of the inscribed pentagon with right angle and dimensions as shown.



(14) Use the given diagram to prove the Pythagorean Theorem.
(Hint: $(a-b)^2 = a^2 - 2ab + b^2$).