**ORMC Beginners 2** **Winter 2026**

# Vectors and Physics 3

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## Warm-up

**Problem 1** *There are three boxes on the table, red, blue, and yellow. One of them has a candy inside, others are empty. It is known that one of the following statements is true while the other two are not.*

1. *The candy is in the red box.*

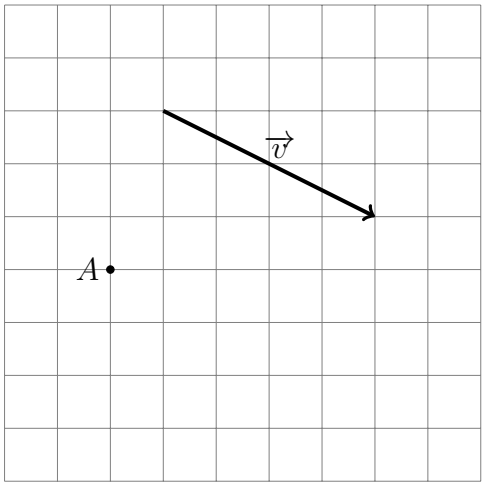
2. *The candy is not in the yellow box.*

3. *The candy is not in the red box.*

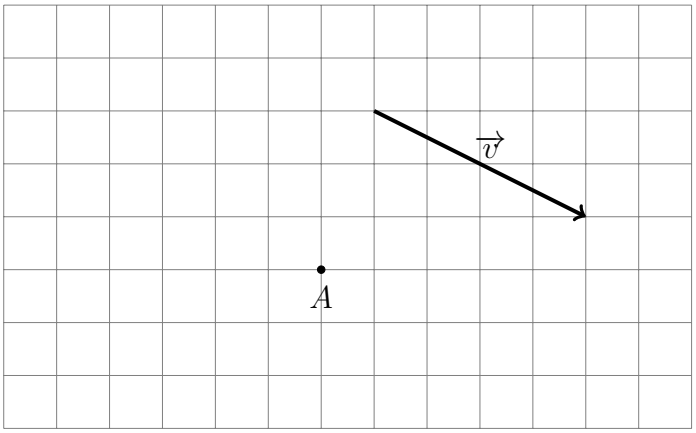
*What box is the candy in?*

## Back to vector algebra

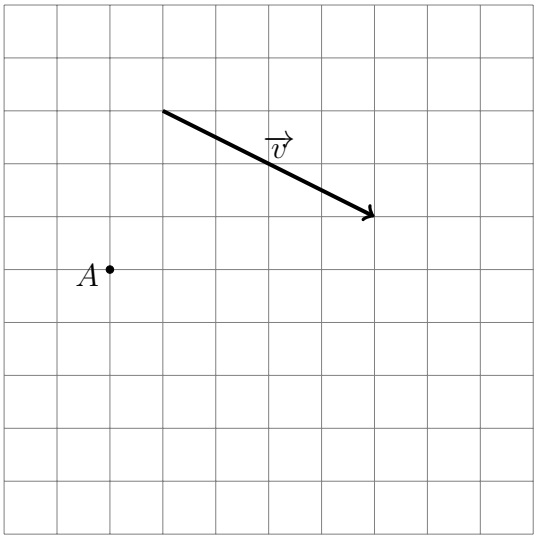
**Problem 2** *For the given vector $\overrightarrow{v}$ and point $A$, construct the vector $\overrightarrow{w} = \overrightarrow{v}$ having $A$ as its initial point on the graph paper below. Use the grid instead of a compass and ruler.*



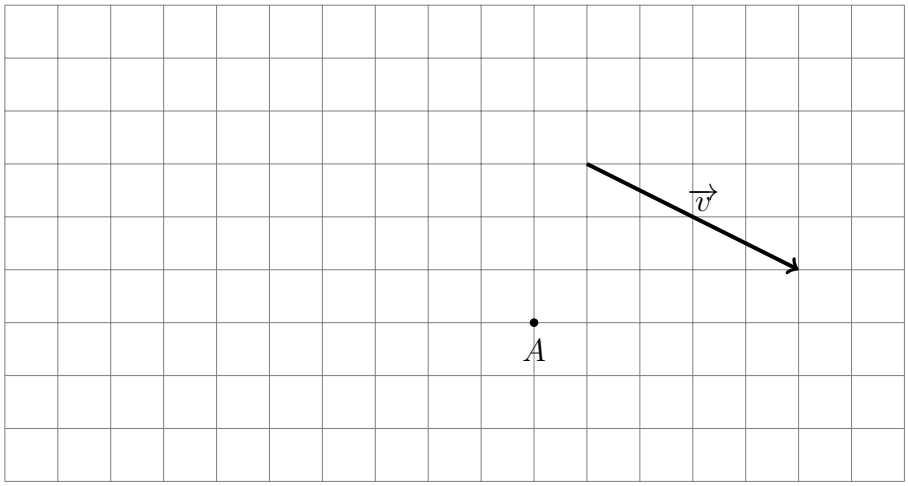

**Problem 3** *For the given vector $\overrightarrow{v}$ and point $A$, construct the vector $\overrightarrow{w} = -\overrightarrow{v}$ having $A$ as its initial point on the graph paper below.*



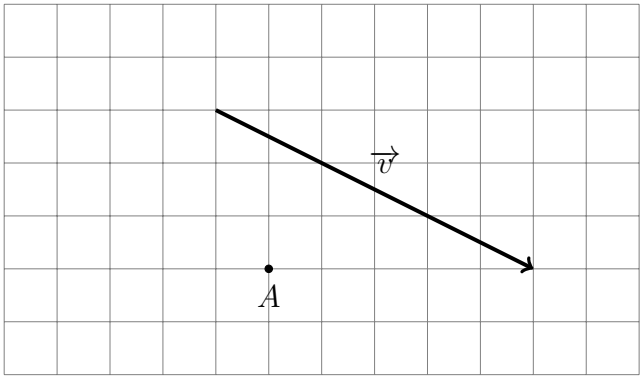
**Problem 4** *For the given vector $\overrightarrow{v}$ and point $A$, construct the vector $\overrightarrow{w} = 1.5\overrightarrow{v}$ having $A$ as its initial point on the graph paper below.*



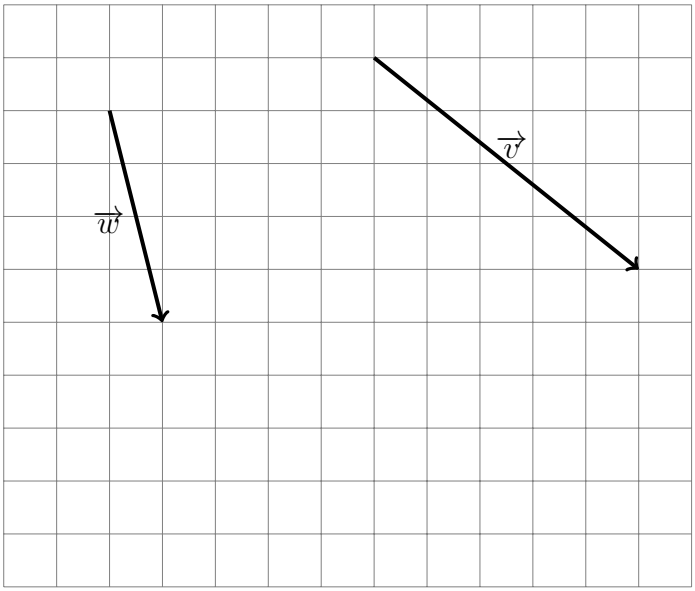

**Problem 5** *For the given vector $\overrightarrow{v}$ and point $A$, construct the vector $\overrightarrow{w} = -2\overrightarrow{v}$ having $A$ as its initial point on the graph paper below.*



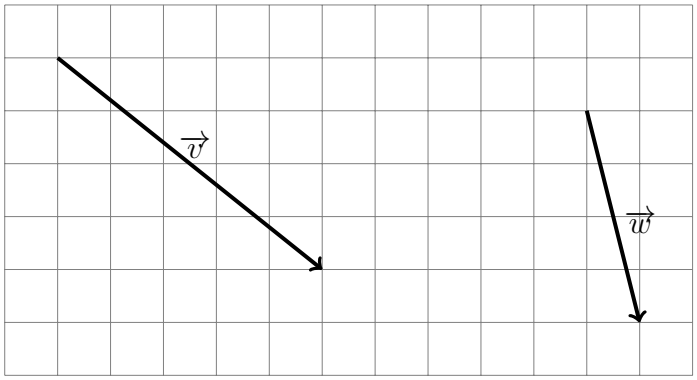
**Problem 6** *For the given vector $\overrightarrow{v}$ and point $A$, construct the vector $\overrightarrow{w} = -\frac{1}{3}\overrightarrow{v}$ having $A$ as its initial point on the graph paper below.*



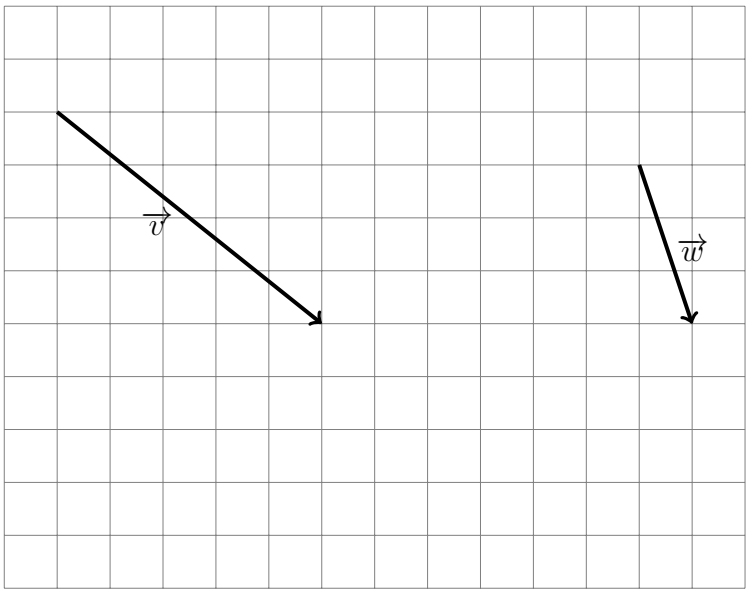

**Problem 7** *For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $\overrightarrow{w} + \overrightarrow{v}$ on the graph paper below.*



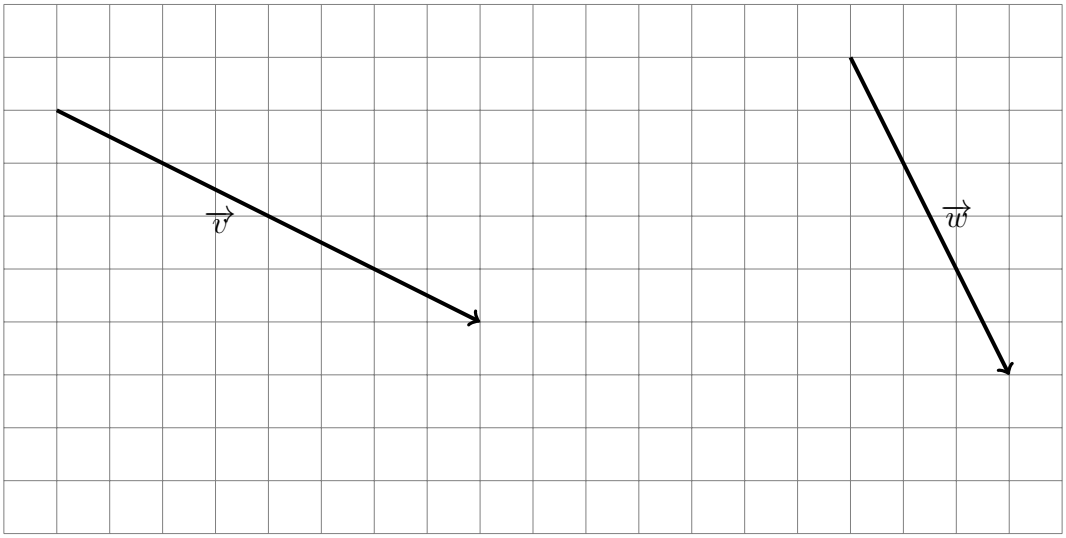
**Problem 8** *For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $\overrightarrow{w} - \overrightarrow{v}$ on the graph paper below.*



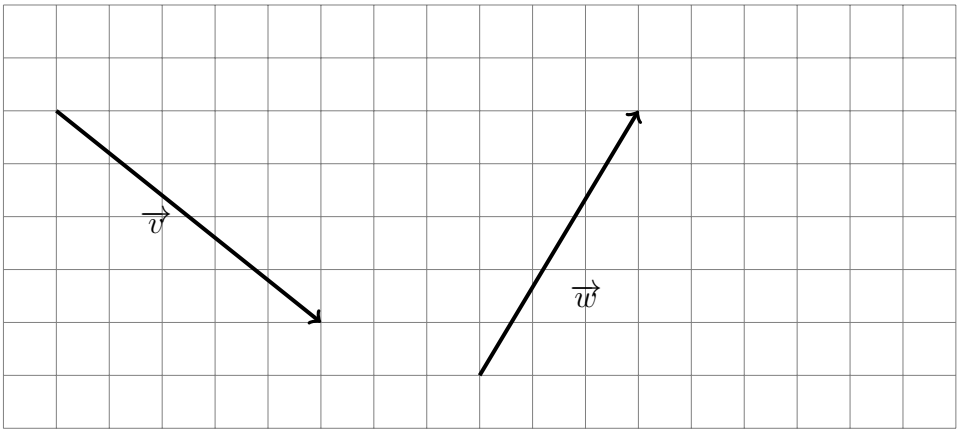

**Problem 9** *For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $2\overrightarrow{v} - 3\overrightarrow{w}$ on the graph paper below.*



**Problem 10** *For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $1.75\overrightarrow{v} - \frac{2}{3}\overrightarrow{w}$ on the graph paper below.*




**Problem 11** *For the given vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, construct the vector $\overrightarrow{v} + \overrightarrow{w}$ originating at the same point as the vector $\overrightarrow{v}$ and the vector $\overrightarrow{w} + \overrightarrow{v}$ originating at the same point as the vector $\overrightarrow{w}$. Is $\overrightarrow{v} + \overrightarrow{w} = \overrightarrow{w} + \overrightarrow{v}$? Why or why not?*

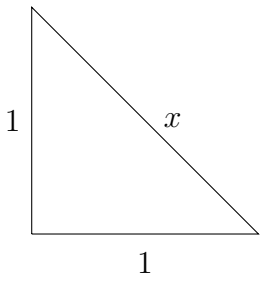


## Pythagoras' theorem and applications

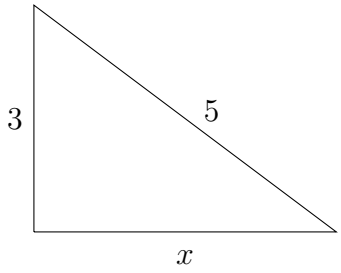
**Problem 12** *Formulate and prove the Pythagoras' theorem.*

**Problem 13** *Use the Pythagors' theorem to find $x$ for the following right triangles.*

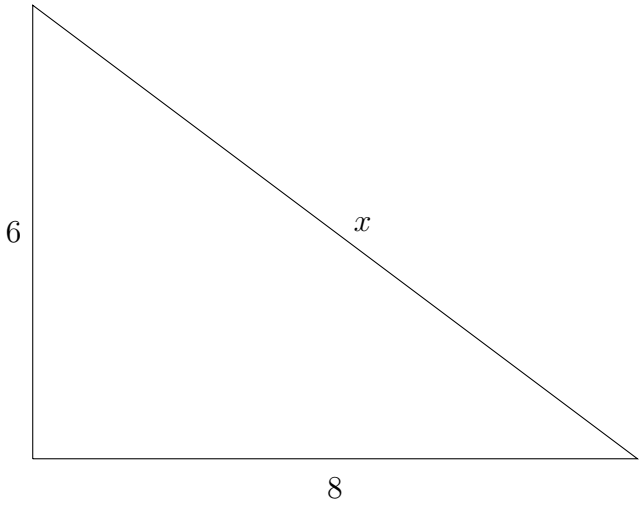
*a.* $x =$



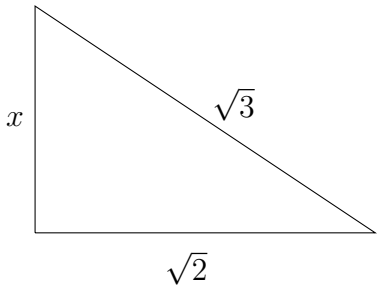

*b.* $x =$



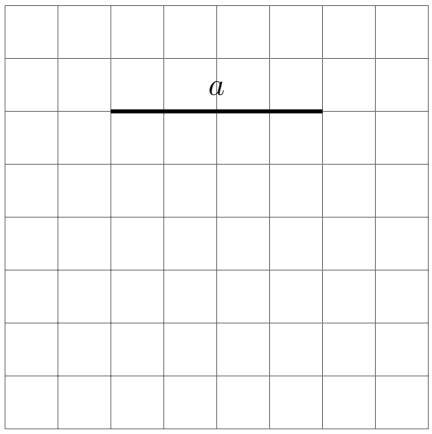

*c.* $x =$



*d.* $x =$




**Problem 14** *Given a segment of length $a$, construct a segment of length $\sqrt{2}a$ on the graph paper below. Use the grid instead of a compass and ruler.*

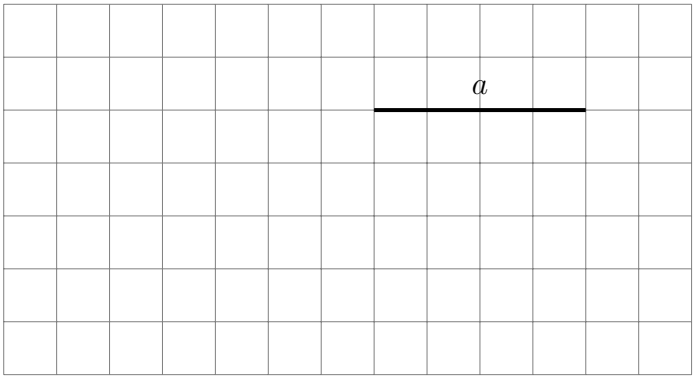


**Problem 15** *Given the segment of length $a$, use a compass and ruler to construct a segment of length $\sqrt{2}a$ in the space below.*



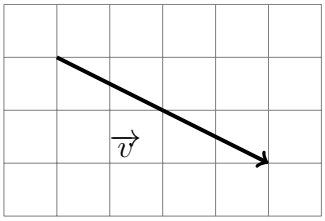

**Problem 16** *Use a compass and ruler to construct a segment of length $\sqrt{3}a$ in the space below where $a$ is the length of the segment from Problem 15.*

**Problem 17** *Given a segment of length $a$, construct a segment of length $\sqrt{5}a$ on the graph paper below. Use the grid instead of a compass and ruler.*

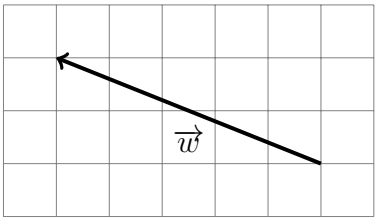



**Problem 18** *Use a compass and ruler to construct a segment of length $\sqrt{6}a$ in the space below where $a$ is the length of the segment from Problem 15.*

**Problem 19** *The side length of a grid square below is one unit. Find the length the vector* $\overrightarrow{v}$*.*

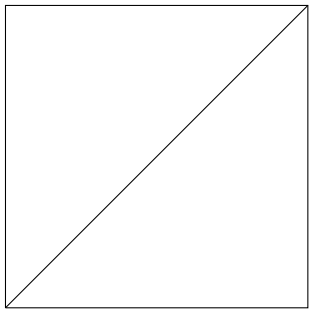



**Problem 20** *The side length of a grid square below is three units. Find the length the vector* $\overrightarrow{w}$*.*



The following problem was communicated to me by one of our students, Ethan Kogan.

**Problem 21** *Prove that the length of the diagonal of a square with the side length one is equal to the square root of two without using the Pythagoras' Theorem.*



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A number is called *rational*, if it can be represented as a ratio $p/q$ of an integer $p$ and a positive integer $q$ such that $p$ and $q$ have no common factors. A number is called *irrational* otherwise.

**Problem 22** *Decide whether the following numbers are rational or irrational. In each case, give a reason.*

*a.* $\dfrac{375}{376}$

*b.* $10$

*c.* $0.5$

*d.* $0$

*e.* $-5$

*f.* $1.2345$

*g.* $0.999999999999999999$

*h.* $\sqrt{2}$

*i.* $\sqrt[3]{10}$

**Problem 23** *Find* $\left\lfloor \sqrt[3]{10} \right\rfloor$ *and* $\left\lceil \sqrt[3]{10} \right\rceil$.

**Problem 24** *Simplify* $\sqrt{8}$.

**Problem 25** *A man is crossing a river in a boat. The speed of the boat is three meters per second. The speed of the water in the river is one meter per second. In what direction should the man steer the boat, if he wants the vessel to move perpendicular to the banks? Please use a compass and a ruler to construct the velocity vector.*

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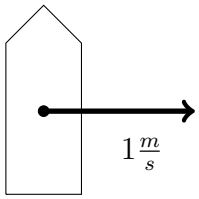


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*The width of the river is* $10\sqrt{2}$ *meters. How long would it take the man to cross the river?*