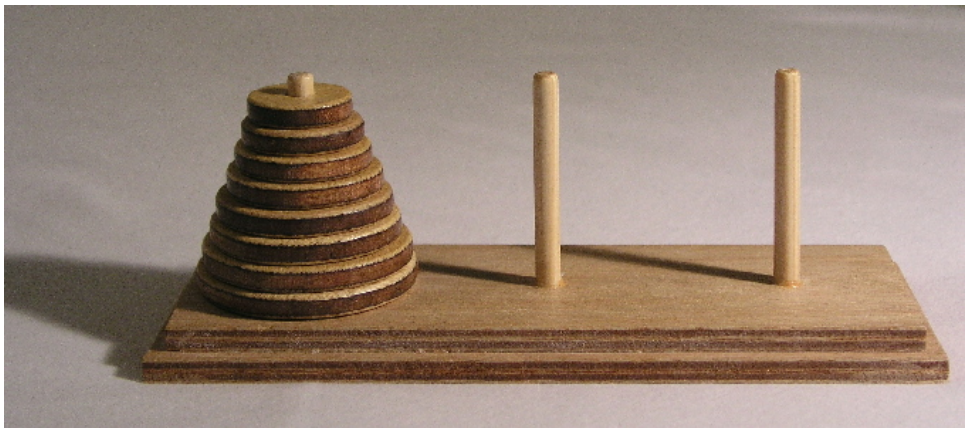
UCLA Olga Radko Endowed Math Circle

Hanoi Tower

Hidden in the jungle near Hanoi, the capital city of Vietnam, there exists a Buddhist monastery where monks keep constantly moving golden disks from one diamond rod to another. There are 64 disks, all of different sizes, and three rods. Only one disk can be moved at a time and no larger disk can be placed on the top of a smaller one. Originally, all the disks were on one rod, say, the left one. At the end, they all must be moved to the right rod. When all the disks are moved, the world will come to an end. (No worries here, it will take the monks a few hundred billion years to complete the task.)



The Hanoi tower with eight disks.

This tale was created by a French mathematician, Édouard Lucas, to promote the puzzle he had invented. Called the Hanoi Tower, it's a great way to get introduced to algorithms.

Definition 1 *An algorithm is a finite set of clear instructions to solve a problem. An algorithm is called optimal, if the solution it provides is the shortest possible.*

Problem 1 *Solve the Hanoi Tower puzzle with two disks. How many steps do you need?*

$$N(2) =$$

Is your algorithm the shortest possible?

Problem 2 *Solve the Hanoi Tower puzzle with three disks. How many steps do you need?*

$$N(3) =$$

Is your algorithm the shortest possible?

Problem 3 *Solve the Hanoi Tower puzzle with four disks. How many steps do you need?*

$$N(4) =$$

Is your algorithm the shortest possible?

Problem 4 *Solve the Hanoi Tower puzzle with five disks. How many steps do you need?*

$$N(5) =$$

Is your algorithm the shortest possible?

Problem 5 *Find the optimal, i.e. shortest, algorithm to solve the Hanoi Tower puzzle with any number of disks.*