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# Intro to Finance (based on Math 174E)

Many of you may eventually work in the finance industry, or at the very least be affected by it in ways beyond our imagination. Today I aim to provide an introduction to that world, based on what I have learned.

The simplest idea of finance is to connect people who seek capital with those willing to take risks. This simple need to connect 2 sides of a contract creates a market, a market trading on contracts, derivatives, ownership, and bets (in some case).

From this simple concept, a whole ecosystem of participants emerges, each with different goals and objectives that complement one another. In most financial markets, there are three primary types of players:

1. **The Speculator:** Participates in the market by betting on the future direction of prices in order to earn profit.

2. **The Hedger:** Participates in the market to reduce uncertainty and manage risk exposure (many examples will follow).

3. **The Arbitrager:** Seeks to earn risk-free profit by exploiting price discrepancies, often by taking offsetting positions between two or more parties.

Here we shall inspect 3 major types of financial contract: The options, the Forward contracts, and the future contracts.

**The Option**: A financial contract that gives its holder the right, ***but not the obligation***, to buy or sell an underlying asset at a predetermined price (strike price) on or before a specified date (expiration date).

The call option typically describes an option giving its holder the right to buy. A long call means you have purchased the right to buy the underlying asset. Short call means you sold an option to buy.

The put option typically describes an option giving its holder the right to sell.

**Problem 1** *Answer the following:*

1. *What is a Long put?*
2. *What is a Short put?*

Stock options are often granted as part of compensation packages. The idea is, with stock option that cannot be exercised until a later date, employees will be motivated to work harder, in hopes of increasing the stock value so they may earn profit on executing the option.

**Problem 2** *Consider the following: Employees are given a stock option as part of their compensation packet. It is a call option, which entails them the right to purchase 1 stock of their company, 1 year from today, at a price of 10 dollars a share.*

1. *In 1 year, if market price is 11 dollars a share, should they execute the contract?*
2. *In 1 year, if market price is 9 dollars a share, should they execute the contract?*

If the market price is 11 in 1 year, then by executing the contract, you are able to purchase 1 stock for 10, and then quickly resell it in the market for 11, essentially netting a profit of 1 dollar.

In the case of market price being 9 in 1 year, if the same actions are taken, a net loss of 1 dollar will be generated. In this case, it would be wiser to not execute the trade and incur no further loss.

Note that typically, if you are to purchase an option in the market, it will cost some money (generally in financial markets, the more freedom you are offered by a contract, the more it will cost).

Here are the payoff graphs for long call and short call respectively.

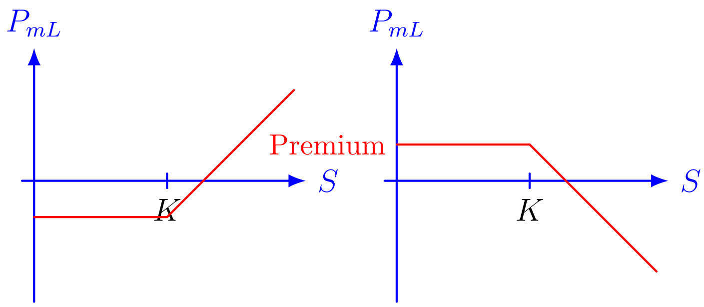



Figure 1: Long and short call payoffs with equal premium magnitude $p$.

K is the strike price, and Profit and Loss (PnL) factors in premium paid or recieved. Notice the symmetry of the payoff graph. If we are to sum up the 2 graphs, we realize the net is zero (aka, if you hold both the long and short position of the same call, you will end up with zero profit).

**Problem 3** *Try to draw the payoff graph for long put and short put. (They will also have the property such that summing them togethere nets zero, so take advantage of that fact)*

In other words, the payoff graphs of Call and Put option suggest that options is fundamentally a zero sum game. Recall the earlier example; for every stock the company sold to their employee at 10 dollars, its one more stock they couldn't sell on the open market at 11 dollars. Essentially, every extra dollar the employee makes off the option is a dollar compensated by the company.

If we connect the idea with our simple definition of finance, roles can be matched. The company is in need of the employees' labor resource, while the employees are willing to take the risk on the performance of the company (in a micro sense, themselves as well).

For speculators and hedgers, similar relations can be established. A hedger may enter a long put position, with the intention that they will be able to sell their goods at strike price, at minimum. A speculator may enter a short put position, with the belief the price will stay above the strike price (thus they can keep the premium without needing to buy).

**Problem 4** *In this pair of speculator and hedger, what resource is provided and what risk is transferred (who is willing to bear risks)?*

This idea of zero sum will return again in other contracts we shall introduce.

**The Forward**: A financial contract that gives its holder the ***obligation*** to buy or sell an underlying asset at a predetermined price (forward price) at a specified future date (maturity date).

The forward contract is simpler; there is only one type of contract., with a long position (obligated to buy), and a short position (obligated to sell). The payoff graph is also much simpler; note the zero sum property as well.

**Problem 5** *Let Y-axis be PnL, and X-axis be market price at date of maturity. Graph the payoff of by both long and short position?*

In the case of forward contract, both parties may act as hedgers or speculators. Note that since there is no freedom to decide at date of maturity, the cost of entering the contract is 0 (it is costless to sign the contract; but you will bear both unlimited loss and unlimited gain, depending on direction of price movement and your position).

**Problem 6** *Suppose that, on May 21, 2020, the treasurer of a U.S. corporation knows that the corporation will pay £1 million in 6 months (i.e., on November 21, 2020). In order to hedge against exchange rate movements, the treasurer decides to enter into a forward contract with an investment bank. The forward price is*

$$F = \$1.2230 \text{ per British pound,}$$

*and the maturity date is November 21, 2020.*

1. *What position in the forward contract should the treasurer take? What is the corresponding position of the investment bank in the forward contract?*

2. *What is the payoff graph of the forward contract in 6 months for both parties?*

3. *What are the payoffs if the GBP/USD exchange rate on November 21, 2020, turns out to be:*

   (a) $S_T = \$1.30$ *per British pound?*

   (b) $S_T = \$1.20$ *per British pound?*

As the previous example have demonstrated, by partaking in a forward contract, one would be able to hedger against the uncertainty price changes. While the treasurer will miss out on any windfall due to price change, they also miss out on any losses due to similar circumstances.

**Problem 7** *Trader A enters into a forward contract to buy an asset for \$1,000 in one year. Trader B buys a call option to buy the asset for \$1,000 in one year. The cost of the option is \$100.*

1. *What is the difference between the positions of the traders? Show the (net) profit and loss (P&L) as a function of the price of the asset in one year for the two traders.*

Notice the difference in losses; while for trader A there is unlimited loss, for trader B they can loss at most 100 dollar (aka the premium paid to enter the options contract).

If we combine different contracts, we are able to achieve some pretty interesting results. Take for example, we entered a long forward with forward price of K, and a long put at strike price of K (with the same expiration date). These positions combined, gives us effectively a long call.

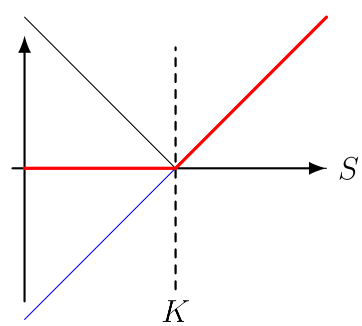



Figure 2: Long forward (blue) plus long put (black). The resulting payoff (red) equals a long call.

Here we will see some more problems, requiring combinations of different contracts.

**Problem 8** *Suppose on May 21, 2020, a corporate treasurer from a U.S. company said: "I will have £1 million to sell in 6 months. If the exchange rate is less than 1.19, I want you to give me 1.19. If it is greater than 1.25, I will accept 1.25. If the exchange rate is between 1.19 and 1.25, I will sell the sterling for the exchange rate." How could you use options written on the GBP/USD exchange rate to satisfy the treasurer?*

**Problem 9** *A stock price is 29 dollar. A trader buys one call option contract on the stock with a strike price of 30 dollar and sells a call option contract on the stock with a strike price of 32.50 dollar. The market prices of the options are 2.75 dollar and 1.50 dollar, respectively. The options have the same maturity date. Describe the trader's position by providing the final payoff function at maturity of the combined position.*

**Problem 10** *Provide and draw the net profit at maturity $T$ as a function of the stock price $S_T$ at time $T$ for the following trading strategies:*

1. ***Butterfly Spread:*** *This position consists of call options with three different strikes but with the same maturity $T > 0$. Specifically, the trader buys two call options with strikes $K_1$ and $K_3$ where $K_3 > K_1$ and sells two calls with strike $K_2 := (K_1 + K_3)/2$.*

2. ***Straddle:*** *The trader has a long position in a call option and a put option for the same stock with the same strike and the same maturity.*

The various strategies adopted by traders can all be drawn out as literal shapes on payout graphs. This is one beautiful thing about visualizing financial concepts. Another great use of these contract is to trade in a risky manner as to increase gains and speculate. Observe the following examples.

**Problem 11** *On May 21, 2020, the market price of Apple stock is \$316.50 and the price of a call option with a strike price of \$320 and a maturity date of September is \$21.70. A trader is considering two alternatives: to buy 100 shares of the stock, or to buy 100 September call options (= 1 call option contract). For each alternative, determine:*

- *a The upfront cost,*
- *b The total gain if the stock price in September is \$400, and*
- *c The total loss if the stock price in September is \$300.*

*Assume that the option is not exercised before September and that positions are unwound at option maturity.*

**Problem 12** *An investor with \$2,000 to invest considers that a stock is likely to increase in value over the next 2 months. The stock price is currently \$20, and a 2-month call option with a \$22.50 strike price is currently selling for \$1. What are possible investment strategies for the speculator? Draw the corresponding investor's profit and loss (P&L) at* $T = 2$ *months as a function of the underlying stock price* $S_T$.

**The Future**: A financial contract that gives its holder the ***obligation*** to buy or sell an underlying asset at a predetermined price (forward price) on or before a specified date (maturity date). The key difference is that futures contracts are adjusted to market price daily through margin accounts (aka mark to market). If margin account falls below maintenance margin, you must refill it up to the initial margin level.

For example, you enter a future contract with a market price of 50000, on the long position. The initial margin is 5000, and the maintenance margin is 2000. On day one, the price of the future contract falls to 45000; your margin account becomes 0, and since it is below the threshold of 2000, you receive a margin call, to refill your margin account up to 5000. The next day, the price of the future raises to 55000; your margin account becomes 15000. You can take out the additional 10000 in your margin account; but note that only 5000 is actual gains, and you've already injected 10000 into the account to begin with.

Such a pattern can go on until the settlement date. Observe the following example.

| day | trade price ($) | settlement price ($) | daily gain ($) | cumulative gain ($) | margin account balance ($) | margin call ($) |
|---|---|---|---|---|---|---|
| 1 | 1,250.00 | | | | 12,000 | |
| 1 | | 1,241.00 | -1,800 | -1,800 | 10,200 | |
| 2 | | 1,238.30 | -540 | -2,340 | 9,660 | |
| 3 | | 1,244.60 | 1,260 | -1,080 | 10,920 | |
| 4 | | 1,241.30 | -660 | -1,740 | 10,260 | |
| 5 | | 1,240.10 | -240 | -1,980 | 10,020 | |
| 6 | | 1,236.20 | -780 | -2,760 | 9,240 | |
| 7 | | 1,229.90 | -1,260 | -4,020 | 7,980 | 4,020 |
| 8 | | 1,230.80 | 180 | -3,840 | 12,180 | |
| 9 | | 1,225.40 | -1,080 | -4,920 | 11,100 | |
| 10 | | 1,228.10 | 540 | -4,380 | 11,640 | |
| 11 | | 1,211.00 | -3,420 | -7,800 | 8,220 | 3,780 |
| 12 | | 1,211.00 | 0 | -7,800 | 12,000 | |
| 13 | | 1,214.30 | 660 | -7,140 | 12,660 | |
| 14 | | 1,216.10 | 360 | -6,780 | 13,020 | |
| 15 | | 1,223.00 | 1,380 | -5,400 | 14,400 | |
| 16 | 1,226.90 | | 780 | -4,620 | 15,180 | |

**Problem 13** *Consider the example above. How many shares are underlying it? What is the net gain or loss based on the difference between initial and settlement date trade price?*

If we consider the day-to-day P&L, let $F_{t,T}$ denote the futures price at time $t$ for a contract maturing at time $T$. For day $t$, the P&L of a long futures position is given by

$$F_{t,T} - F_{t-1,T}.$$

Summing over all days from $t = 1$ to $t = T$, we obtain

$$F_{T,T} - F_{0,T} = \sum_{i=1}^{T} \left(F_{i,T} - F_{i-1,T}\right).$$

The exact same idea carries over to short futures position as well (recall the zero sum concept from earlier; what would the equation look like). Therefore, if one is to hold the future contract fully from 1st day til the maturity day, the net PnL should be identical to a forward contract with the same strike price and same maturity date.

Now let us investigate more deeply the idea of arbitrage. In the general sense, arbitrage means taking advantage of price differences. In the specific sense of finance, for an opportunity to be called an arbitrage, it must satisfy two conditions:

1. No set-up cost.
2. Guaranteed profit.

Most arbitrage opportunities do not persist for long; as more arbitrageurs discover them, their action directly removes its existence. Many scams are based on advertising arbitrage opportunities (research the infamous Ponzi scheme; what is the arbitrage opportunity advertised).

**Problem 14** *Explain the arbitrage opportunity (price arbitrage) when the price of a dually listed mining company stock is 50 (USD) on the New York Stock Exchange and 60 (CAD) on the Toronto Stock Exchange. Assume that the exchange rate is such that 1 U.S. dollar equals 1.21 Canadian dollars.*

Since arbitrage opportunities are quickly exploited and eliminated, prices must satisfy certain consistency conditions. Take for example, the price of premium for a call option. Suppose premium equals market price at day of, aka

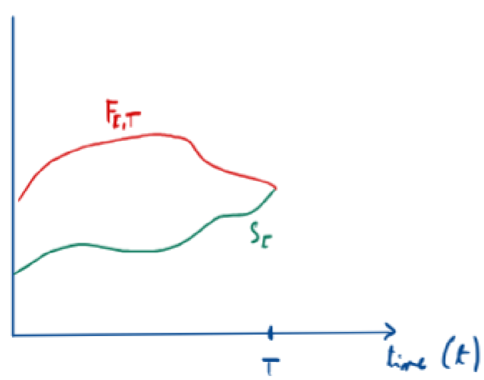
$$C_0(K,T) = S_0$$

Then we can set up the following arbitrage:

| | $t=0$ | $t=T$ | |
|---|---|---|---|
| | | $S_T > K$ | $S_T < K$ |
| sell call (short position) | $-C_0(K,T)$ | $-(S_T-K)$ | 0 |
| buy stock (long position) | $+S_0$ | $+S_T$ | $+S_T$ |
| net value | 0 | $+K$ | $+S_T$ |

**Problem 15** *Consider our more strict definition of arbitrage. Does this scenario fits?*

Several important results follow from the principle of no arbitrage. In particular, consider the relationship between the market price and the futures price on the maturity date. The following graph illustrates the futures–spot convergence property: as the contract approaches its maturity date T, the futures price specified in the contract converges to the prevailing market price. At maturity, the futures price equals the spot price.



**Problem 16** *Suppose this convergence does not hold. What kind of arbitrage opportunities can be constructed? Illustrate your answer with a graph similar to the premium pricing argument.*

1. $F_{T,T} > S_T$,
2. $F_{T,T} < S_T$.

Finally, we will observe some hedging strategy. The futures market is often used as an instrument of hedging, as relatively speaking it is much easier to enter and exit a hedge contract (simply need to settle today's' account and then take on an opposing position), and costless to set up.

Suppose you are obligated to sell oil at market price in 1 year. You can hedge against the future act of selling by shorting first, and then either

1. Hold and execute both contracts at the day of maturity,
2. Settle the short position using a long, and execute just the original contract at day of.

The following is the payoff of holding and executing both contract at day of maturity (assume everything converts to cash, and we fulfill contracts by purchasing at the day of)

| Time | 0 | 3M |
|---|---|---|
| Position | $-F_{3M}$ | 0 |
| cash | 0 | $S_{3M} + F_{3M} - S_{3M}$ $= F_{3M}$. |

**Problem 17** *What is the payoff using option 2?*

**Problem 18** *Suppose I am trying to set up a hedge, and I am currently in a contract to buy oil at time = T. What position should I take up? What is the pay off at the end?*

The effective price consists of two components: a fixed component $F_{0,T}$, known at time 0, and a basis component $S_T - F_{T,T}$, which is uncertain until maturity.

Generally, we settle with the following price at time = T:

$$F_{t_1} + (S_{t_2} - F_{t_2})$$

If you are a seller, you receive payment of $F_{t_1} + (S_{t_2} - F_{t_2})$; as a buyer, you will pay out $F_{t_1} + (S_{t_2} - F_{t_2})$. Ideally, at time = T, by no-arbitrage (or any arbitrage opportunity would have been taken already), the futures price must equal the spot price at maturity. However since we do not wish to risk the possibility of delivery, we will close out a bit earlier. The uncertainty part of $(S_{t_2} - F_{t_2})$ is known as the basis.

**Problem 19** *Consider $S_{t_2} > F_{t_2}$. If you are a seller, how does this impact your hedger? What about if you are a buyer?*

We will close the packet with introduction into interest, present value, and future value. The simplest idea of present value and future value is to imagine saving 100 dollars in a bank. If in a bank, you will receive a risk free interest rate; depending on the frequency of compounding and rate itself, your savings after one year will grow according to the compounding formula.

Here is the formula: let m = number of compounding periods per year, r = rate, and T = time intended to save.

$$FV = PV\left(1 + \frac{r}{m}\right)^{mT}$$

If the compound is continuous, the formula becomes:

$$FV = PV\, e^{rT}$$

Present value works on a similar principle; suppose your bank promises to pay 4 dollars every half year, and interest is compounded continuously, and you intend to withdraw in 2 years. Then the PV of your savings becomes.

$$PV = 4e^{-0.5r} + 4e^{-1.0r} + 4e^{-1.5r} + 104e^{-2.0r}$$

**Problem 20** *Today (at time $t = 0$), suppose that a 2-year bond with a principal of $100 provides coupons at the rate of 6% per annum, paid semi-annually.*

*The following are today's market spot rates:*

| ***Maturity (years)*** | ***Spot rate p.a. (%) (cont. comp.)*** |
|---|---|
| *0.5* | *5.0* |
| *1.0* | *5.8* |
| *1.5* | *6.4* |
| *2.0* | *6.8* |

*Today's theoretical price (at time $t = 0$) of the bond can be computed as:*