Meeting 5: Probability

2/10/13

$$\text{Probability} = \frac{number\ \ of\ \ successful\ \ outcomes}{number\ \ of\ \ total\ \ outcomes}$$

1. We have a standard coin, one side is called "heads" (H) and the other side "tails" (T). Using the definition of probabiliy given above, answer the following questions:

   (a) List all possible outcomes if you were to flip the coin once.

   (b) What is the probability the coin will land on H?

   (c) What is the probability the coin will land on T?

   (d) What is the probability the coin will land on either H or T?

(e) Add your answer from (b) and (c) together. What do you notice about the answer compared to part (d)?

2. Now let's flip two coins at the same time.

(a) List all possible outcomes below.

(b) What is the probability of getting HH?

(c) What is the probability of getting one H and one T? (the order doesn't matter, so count both TH and HT as the same)

(d) What is the probability of getting TT?

(e) If you flip 2 coins 400 times, approximately how many times do you expect to get HH?

Approximately how many times do you expect to get one H and one T? (the order doesn't matter)

3. Now we flip 3 coins at the same time.

(a) What are all the possible outcomes?

(b) What is the probability of getting HHH?

(c) What is the probability of getting one T and two Hs? (the order doesn't matter)? Start by listing all possible outcomes with one T and two Hs:

Probability(THH) =

(d) What is the probability of getting one H and two Ts (the order doesn't matter)? Start by listing all the possible combinations:

Probability(HTT) =

(e) What is the probability of getting TTT?

(f) What is the probability of landing on either T or H for all three coins?

Add the probabilities from parts (b), (c), (d), and (e) and compare it to your answer from (f).

4. We roll a single die.

   (a) What are all the possible outcomes of the roll?

   (b) What is the probability the outcome will be 3?

   (c) What is the probability the outcome will be 4?

   (d) If I roll only once, are all outcomes equally possible or are some more likely than others?

   (e) What is the probability of the outcome being an even number?

(f) What is the probability of the outcome being an odd number?

5. We color the faces of a single die: 4 faces are red and 2 faces are blue.

(a) Olga says that since we colored the die with two colors, it follows that the probability of rolling a red face is $\frac{1}{2}$ and the probability of rolling a blue face is $\frac{1}{2}$. Is she correct? Why or why not?

(b) What are the correct probabilities for rolling a red face and rolling a blue face?

Probability(Red) =

Probability(Blue) =

6. Now we roll 2 dice at the same time. (Like we did two lessons ago, but we do not add up the numbers!)

   (a) The outcomes can be listed as pairs of numbers (x, y):
   (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), ..., ..., (6, 4), (6, 5), (6, 6)

   How many possible outcomes are there?

   (b) What is the probability of the outcome (1, 2)?

   (c) What is the probability that the outcome will be (3, 5)?

(d) What is the probability that both rolls give the same number? First, shade the outcomes that contribute to this event in the table below:

	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Probability(x=y) =

(e) What is the probability that $x \leq 3$ (i.e., the first number is less than or equal to 3)? First, shade in the outcomes that apply in the table below:

	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Probability($x \leq 3$) =

(f) What is the probability of the event that $y > x$ (i.e., the number on the second die is strictly larger than the number on the first die)?

	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Probability($y > x$) =

(g) What is the probability the first number is an odd number? First, shade and count the appropriate outcomes.

	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Probability(x is odd) =

(h) What is the probability that both numbers are even?

	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Probability(x is even and y is even) =