

GEOMETRY - FINDING ANGLES II

MATH CIRCLE (INTERMEDIATE) 03/03/2013

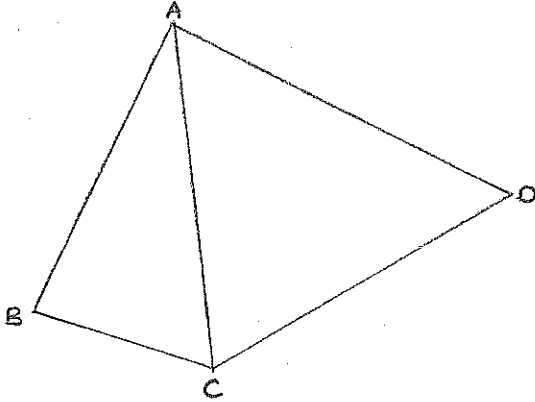
Facts about parallel lines:

When two parallel lines are cut by a transversal (see figure; line m is the transversal), the following sets of angles are congruent:

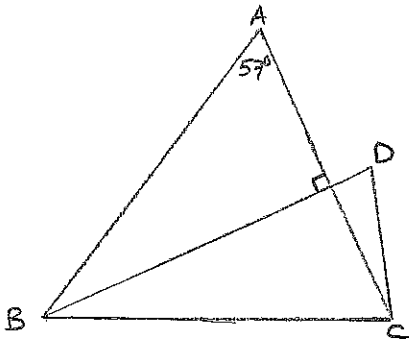
- Alternate interior angles (for example, α and β)
- Vertical angles (for example, γ and α)
- Corresponding angles (for example, γ and β)

- (1) Let's prove that the sum of the angles in a triangle is always 180 degrees.
- (a) Draw a triangle, any triangle. It does not need to be equilateral or even isosceles. Label the vertices as A , B , and C .
- (b) Draw on the above diagram a line l that passes through point C and is parallel to segment AB .
- (c) Consider the three acute angles formed by the intersection of line l with triangle ABC . What is the sum of these angles' measures?
- (d) Use your result from part (c), and the facts about parallel lines cut by a transversal above, to prove that the sum of the measures of the interior angles of triangle ABC is 180 degrees.

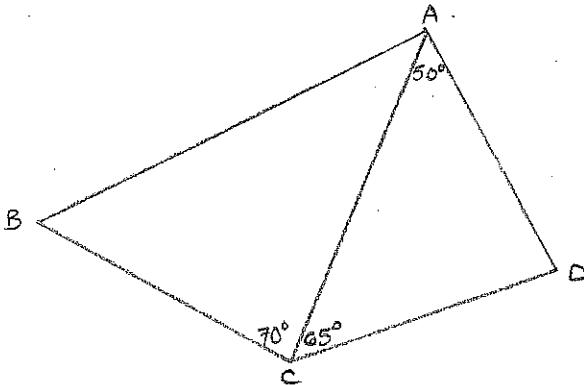
- (2) In the picture, $AD = DC$, $AB = AC$, and the measures of angles ABC and ADC are 75 and 50 degrees, respectively. What is the measure of angle BAD ?



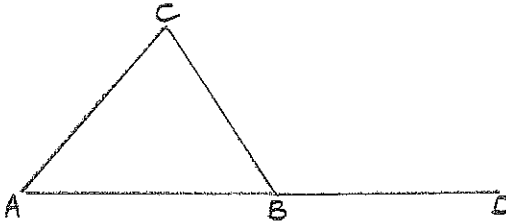
- (3) ABC is an isosceles triangle such that $AC = BC$. CBD is an isosceles triangle such that $CB = DB$. BD meets AC at a right angle. If the measure of angle A is 57 degrees, what is the measure of angle D ?



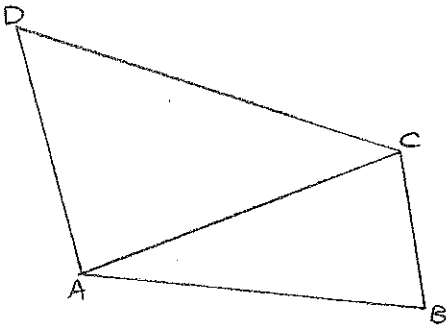
- (4) Some angle measures of the quadrilateral $ABCD$ are shown in the picture to the right. In addition, $AD = BC$. Find the measure of angle ABC .



- (5) Triangle ABC is equilateral. Point B is the midpoint of segment AD . Point E belongs to a circle which has a center at point D and whose radius equals AB . What is the measure of angle BED when the distance CE is the greatest possible?

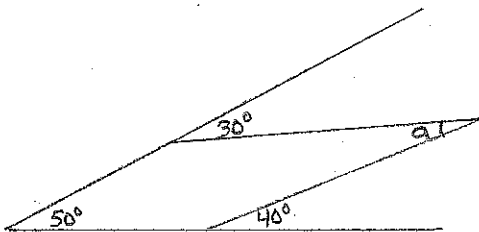


- (6) In the convex quadrilateral $ABCD$, $AB = AC$. In addition, $m\angle BAD = 80^\circ$, $m\angle ABC = 75^\circ$, and $m\angle ADC = 65^\circ$. What is the measure of angle BDC ?



(Not drawn to scale)

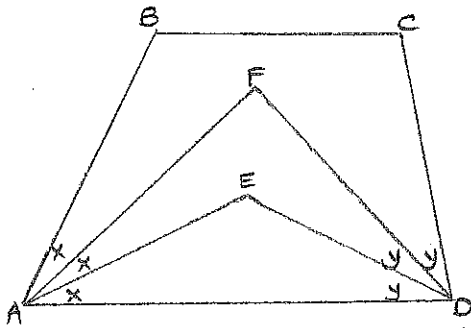
- (7) What is the measure of angle a in the figure?



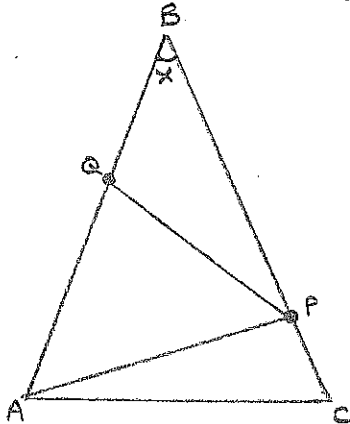
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- (8) In triangle ABC , the bisectors of angles ABC and ACB intersect at point D . We know that the measure of angle BDC is 150 degrees. What is the measure of angle BAC ?

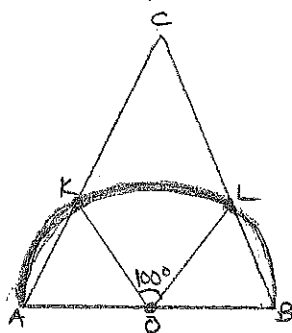
- (9) In quadrilateral $ABCD$, $\angle ABC = 110^\circ$, $\angle BCD = 100^\circ$, and angles BAD and CDA are trisected as shown. What is the degree measure of $\angle AFD$?



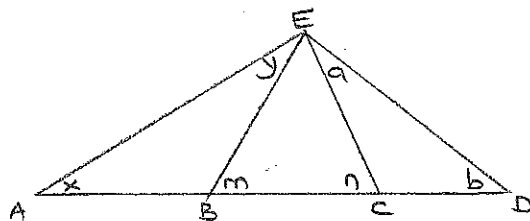
- (10) Triangle ABC is isosceles with base AC . Points P and Q are respectively on CB and AB such that $AC = AP = PQ = QB$. Find the measure of $\angle B$.



- (11) The diameter of the circle with center O is also the base of the isosceles triangle ABC . Points K and L are points of intersection of the triangle with the circle. If $\angle KOL = 100^\circ$, what is the measure of angle C of triangle ABC ?

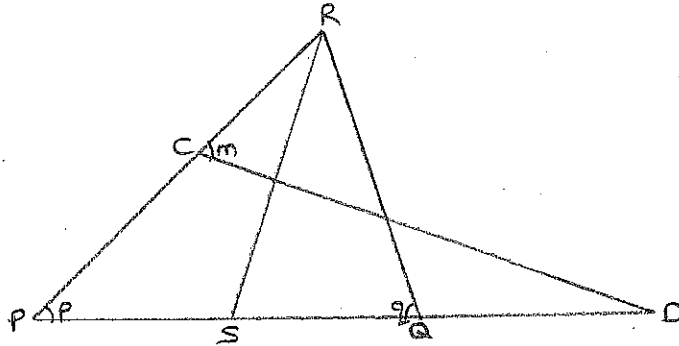


- (12) In a general triangle ADE (as shown) lines EB and EC are drawn.



- (a) Show that $x + y = m$. (This convenient result is called the **Exterior Angle Theorem**).
- (b) Show that $x + y + n = a + b + m$.

- (13) Given triangle PQR with RS bisecting $\angle R$, PQ extended to D , and CD perpendicular to RS , show that $m = \frac{p+q}{2}$.



HINTS:**Problem 2.**

Hint 2.1: If two sides of a triangle are congruent, the angles opposite those sides are also congruent.

Hint 2.2: What is the sum of the measures of the angles of a triangle?

Problem 3.

Hint 3.1: See Hint 2.1.

Problem 4.

Hint 4.1: See Hint 2.2.

Hint 4.2: Hint 2.1 goes the other way, too: if two angles of a triangle are congruent, then the sides opposite those angles are also congruent.

Problem 5.

Hint 5.1: Complete the figure by drawing the specified circle. Label point E on the circle and draw segment BE.

Hint 5.2: What is the measure of each angle in an equilateral triangle?

Hint 5.3: What is the sum of the measures of the angles that form a straight line?

Problem 6.

Hint 6.1: What is the sum of the measures of the angles of a quadrilateral?

Hint 6.2: See Hint 4.2.

Problem 8.

Hint 8.1: Let a be the measure of one of the angles formed by the bisector of angle ABC . Let b be the measure of one of the angles formed by the bisector of angle ACB . What is the value of $a + b$?

Hint 8.2: What is the measure of angle BAC in terms of a and b ? Now apply the result of Hint 8.1.

Problem 9.

Hint 9.1: See Hint 6.1.

Hint 9.2: Use a similar approach to Problem 8 (see Hint 8.1). Can you express the measure of $\angle AFD$ in terms of x and y ?

Problem 10.

Hint 10.1: Let x equal the measure of $\angle B$. What is the measure of angle PQA in terms of x ? $\angle APC$? $\angle BCA$? (Use Hints 2.1 and the result of Problem 1.)

Problem 11.

Hint 11.1: Can you find any isosceles triangles in the figure?

