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Warm-up

Problem 1 *Jane is six times younger than her great grandfather. She also has noticed that if she inserts a zero between the digits of the number showing her age, she will get that of her great granddad. How old is Jane?*

Problem 2 *Two travellers walked towards each other along a road connecting the cities A and B. They started out simultaneously and walked at constant, but different, speeds. The moment they met, the first traveller needed 16 hours to complete the journey while the second needed nine. How much time did it take them to meet on the road?*

Problem 3 *100 people took part in a shooting competition. The first guy to shoot scored 80 points. The second scored 60. The third shooter got the average score of the first two. The fourth competitor got the average score of the first three and so on it went. Finally, the last shooter got the average score of the previous 99 folks. What was his score?*

Problem 4 *Check that $(3^2 + 5^2)^2 = 16^2 + 30^2$. Prove that no matter what natural numbers a and b you take, you can always find natural numbers x and y such that $(a^2 + b^2)^2 = x^2 + y^2$.*

Problem 5 Put the numbers 2^{45} , 3^{36} , 4^{27} , and 5^{18} in the increasing order.

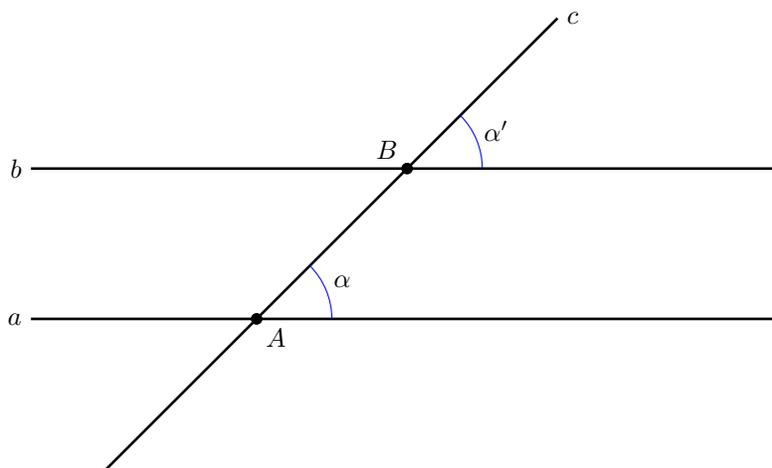
Problem 6 How many full circles does the bisector of the angle between the hour and minute hands of a regular (not 24 hour) clock make in one day?

Back to Geometry

Proposition 1 *An external angle of a triangle is greater than either of the two angles of the triangle it is not supplementary to.*

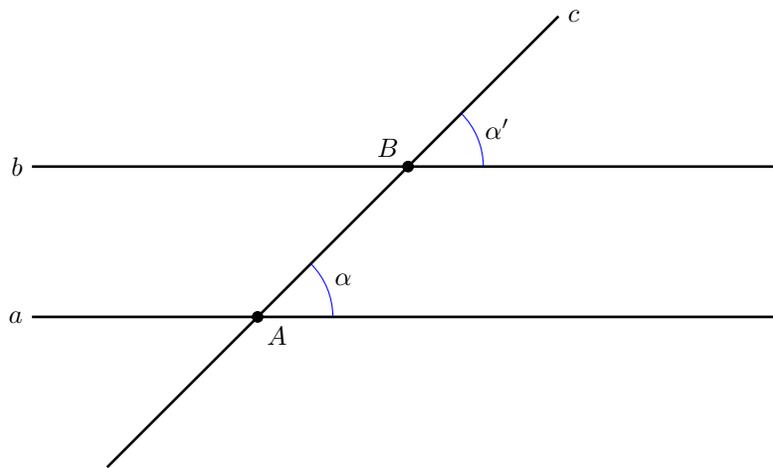
Problem 7 *Prove Proposition 1.*

Proposition 2 *If two distinct straight lines in the Euclidean plane form the angles of equal size with a third straight line in the plane, then they are parallel.*



Problem 8 *Prove Proposition 2.*

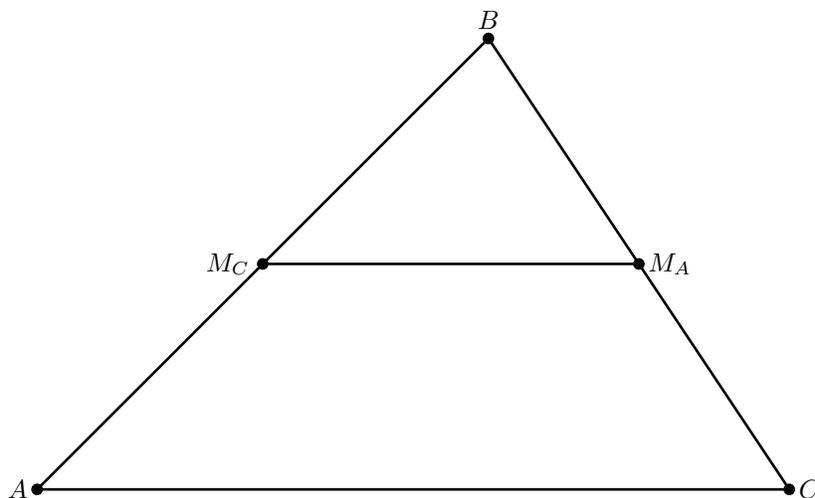
Proposition 3 *If two distinct straight lines in the Euclidean plane are parallel, then they form angles of equal size with a third straight line crossing them.*



Note 1 *Proposition 3 is opposite to Proposition 2.*

Problem 9 *Prove Proposition 3.*

Problem 10 *Prove that a segment of the straight line connecting the midpoints of two sides of a triangle in the Euclidean plane is parallel to the third side and has a half of its length.*

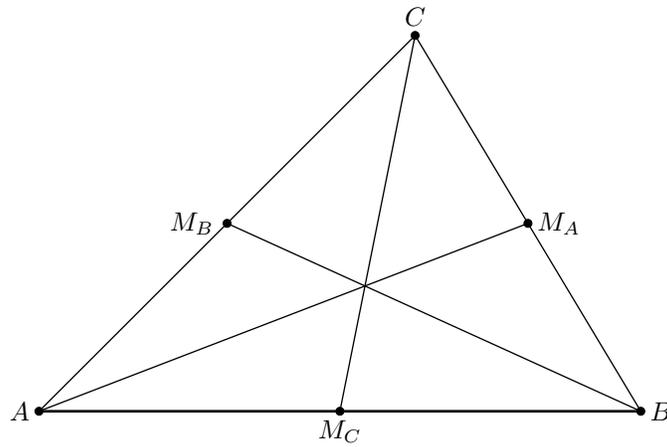


Note 2 *The statement of Problem 10 can be easily generalized. If*

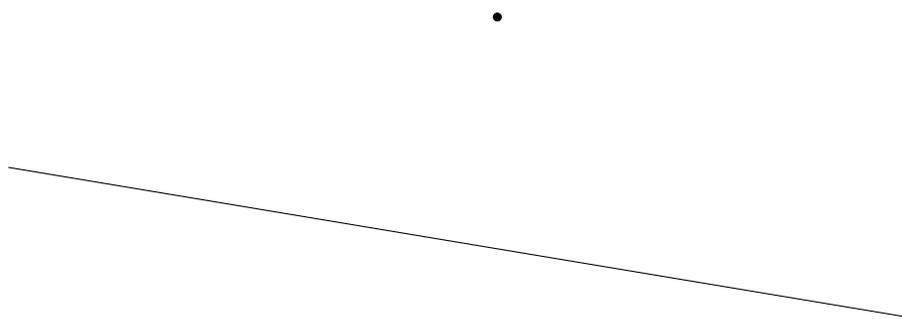
$$\frac{|BM_C|}{|M_CA|} = \frac{|BM_A|}{|M_AC|} = \frac{p}{q},$$

then the line M_CM_A is parallel to the line AC and $|AC| = (1 + q/p)|M_CM_A|$.

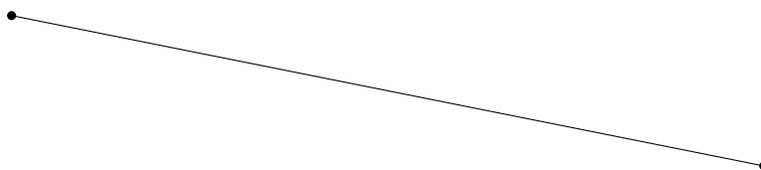
Problem 11 *Prove that the medians of a triangle intersect at one point. The point divides each median in the ratio 2 : 1 counting from the vertex.*



Problem 12 Use a compass and a ruler to draw a straight line parallel to the given one and passing through the given point below.



Problem 13 Use a compass and a ruler to divide the segment below into three parts of equal length.



Problem 14 *Prove that the angles of any triangle in the Euclidean plane always add up to a straight angle.*

A bit of some funny physics here...

Problem 15 *The more it dries, the more wet it becomes. What is it?*